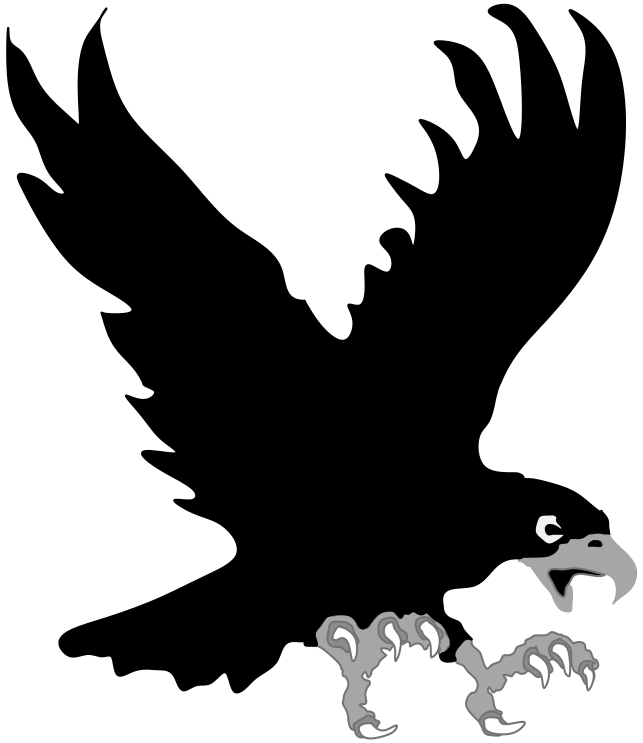


“Temel Matematiksel Kavramlar”

Dr. Cahit Karakuş, 2020

Uygulamalı Matematik

- Türevin geometrik yorumu
- Eğim
- Limit
- İntegral
- Ayrık matematik
- Diferansiyel denklemler
- Nümerik analiz
- Matlab
- Nonlineer dinamik
- Sinyal analizi

Birimler

Common Powers

Prefix	Symbol	Power of 10	Power of 2	Prefix	Symbol	Power of 10
Kilo	K	1 thousand = 10^3	$2^{10} = 1024$	Milli	m	1 thousandth = 10^{-3}
Mega	M	1 million = 10^6	2^{20}	Micro	μ	1 millionth = 10^{-6}
Giga	G	1 billion = 10^9	2^{30}	Nano	n	1 billionth = 10^{-9}
Tera	T	1 trillion = 10^{12}	2^{40}	Pico	p	1 trillionth = 10^{-12}
Peta	P	1 quadrillion = 10^{15}	2^{50}	Femto	f	1 quadrillionth = 10^{-15}
Exa	E	1 quintillion = 10^{18}	2^{60}	Atto	a	1 quintillionth = 10^{-18}
Zetta	Z	1 sextillion = 10^{21}	2^{70}	Zepto	z	1 sextillionth = 10^{-21}
Yotta	Y	1 septillion = 10^{24}	2^{80}	Yocto	y	1 septillionth = 10^{-24}

UNITS OF LENGTH

1 inch (in)	=	2.54 centimeters (cm)
1 foot (ft)	=	30.48 cm = 0.3048 m
1 yard (yd)	≅	0.9144 meter
1 meter (m)	≅	39.37 inches
1 kilometer (km)	≅	0.54 nautical mile
	≅	0.62 statute mile
	≅	1093.6 yards
	≅	3280.8 feet
1 statute mile	≅	0.87 nautical mile
(sm or stat. mile)	≅	1.61 kilometers
	=	1760 yards
	=	5280 feet
1 nautical mile	≅	1.15 statute miles
(nm or naut. mile)	≅	1.852 kilometers
	≅	2025 yards
	≅	6076 feet
1 furlong	=	1/8 mi (220 yds)

UNITS OF SPEED

1 foot/sec (fps)	≅	0.59 knot (kt)*
	≅	0.68 stat. mph
	≅	1.1 kilometers/hr
1000 fps	≅	600 knots
1 kilometer/hr	≅	0.54 knot
(km/hr)	≅	0.62 stat. mph
	≅	0.91 ft/sec
1 mile/hr (stat.)	≅	0.87 knot
(mph)	≅	1.61 kilometers/hr
	≅	1.47 ft/sec
1 knot*	≅	1.15 stat. mph
	≅	1.69 feet/sec
	≅	1.85 kilometer/hr
	≅	0.515 m/sec

*A knot is 1 nautical mile per hour.

Units		Inches	Feet	Yards	Miles	Centimeters	Meters
1 inch	=	1	0.083 333 33	0.027 777 78	0.000 015 782 83	2.54	0.025 4
1 foot	=	12	1	0.333 333 3	0.000 189 393 9	30.48	0.304 8
1 yard	=	36	3	1	0.000 568 181 8	91.44	0.914 4
1 mile	=	63 360	5 280	1 760	1	160 934.4	1609.344
1 centimeter	=	0.393 700 8	0.032 808 40	0.010 936 13	0.000 006 213 712	1	0.01
1 meter	=	39.370 08	3.280 840	1.093 613	0.000 621 371 2	100	1

UNITS OF VOLUME

1 gallon	≅	3.78 liters
	≅	231 cubic inches
	≅	0.1335 cubic ft
	≅	4 quarts
	≅	8 pints
1 fl ounce	≅	29.57 cubic centimeter (cc) or milliliters (ml)
1 in^3	≅	16.387 cc

UNITS OF AREA

1 sq meter	≅	10.76 sq ft
1 sq in	≅	645 sq millimeters (mm)
	=	1,000,000 sq mil
1 mil	=	0.001 inch
1 acre	=	43,560 sq ft

UNITS OF WEIGHT

1 kilogram (kg)	≅	2.2 pounds (lbs)
1 pound	≅	0.45 Kg
	=	16 ounce (oz)
1 oz	=	437.5 grains
1 carat	≅	200 mg
1 stone (U.K.)	≅	6.36 kg

NOTE: These are the U.S. customary (avoirdupois) equivalents, the troy or apothecary system of equivalents, which differ markedly, was used long ago by pharmacists.

UNITS OF POWER / ENERGY

1 H.P.	=	33,000 ft-lbs/min
	=	550 ft-lbs/sec
	≅	746 Watts
	≅	2,545 BTU/hr

(BTU = British Thermal Unit)

1 BTU	≅	1055 Joules
	≅	778 ft-lbs
	≅	0.293 Watt-hrs

ENERGY CONVERSIONS

1 BARREL OF OIL

= 5.8 X 10^6 BTU

= 42 US gallons = approx. 159 litres

1 cubic metre = 35.315 cubic feet = 6.2898 barrels

1 tonne of crude oil = approx. 7.3 barrels

Tonne of oil equivalent

The **tonne of oil equivalent** (**toe**) is a unit of energy defined as the amount of energy released by burning one tonne of crude oil.

Mtoe, one million toe

gigatoe (Gtoe, one billion toe).

A smaller unit of **kilogram of oil equivalent** (**kgoe**) is also sometimes used denoting 1/1000 toe.

- 1 toe = 39,683,205.411 BTU
- 1 toe = 7.11, 7.33, or 7.4 barrel of oil equivalent (boe)
- 1 barrel of oil equivalent (boe) contains approximately 0.146 toe (i.e. there are approximately 6.841 boe in a toe).

ROMA RAKAMLARI

Sembol	**Name**	**Değer**	**Tanım**
I	*unus*	1	Bir
V	*quinque*	5	Beş
X	*decem*	10	On
L	*quinquaginta*	50	Elli
C	*centum*	100	Yüz
D	*quingenti*	500	Beş yüz
M	*mille*	1,000	Bin

$\overline{V}$ (5,000) $\overline{X}$(10,000) $\overline{L}$(50,000) $\overline{C}$(100,000) $\overline{D}$(500,000) $\overline{M}$(1,000,000)

$\overline{CMXXIV}$DLXXXVII (924,587)

MDCCCLXXXVIII is 1000+500+100+100+100+50+10+10+10+5+1+1+1 or 1888
MCMXCIX is M CM XC IX or 1000+(1000-100)+(100-10)+(10-1) or 1999

Complexs Numbers

Complex numbers

- A complex number $Z = \epsilon C$ is of the form $a, b = \epsilon R$ where $Z = a + ib$ **and *i*^{2}=-*1***
- Polar representation $Z = Ue^{i\theta}$, where U,θ ϵR

  - *With* $U = \sqrt{a^2 + b^2}$ *the modulus or magnitude*

  - *And the phase* $\theta = \arctan(b/a)$; a ve b'nin işaretlerine bakılarak açının hangi düzlemde olduğu belirlenir (a,b):(+,+),(-,+), (-,-), (+,-).

- *Complex* ***conjugate***

$$Z = U(\cos\theta + \mathrm{i}\sin\theta) = Ue^{i\theta}$$

$$Z^* = (a + ib\ )^* = Ue^{-i\theta} = U(\cos\theta - \mathrm{i}\sin\theta) = a - ib$$

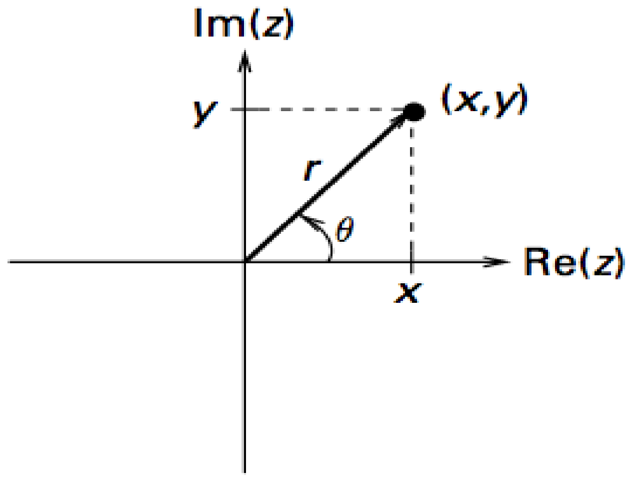
Complex numbers

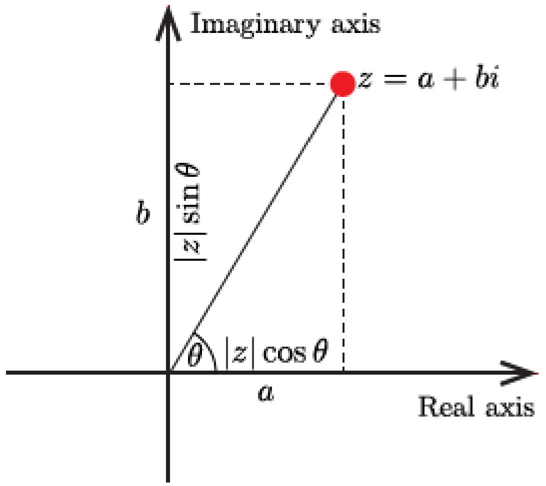
- Complex numbers provide a compact way of describing amplitude and phase (and the operations that affect them, such as filtering)

Complex number $z = x + jy$ (x and y real-valued $j = \sqrt{-1}$.)

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$r = |z| = \sqrt{x^2 + y^2},$$
$$\theta = \arg(z) = \tan^{-1}\frac{y}{x}$$





Complex Numbers Properties

1. $z+w=w+z$
2. $zw=wz$
3. $\overline{z+w}=\bar{z}+\bar{w}$
4. $\overline{zw}=\bar{z}\bar{w}$
5. $z\bar{z}=\bar{z}z=|z|^2$
6. $\bar{\bar{z}}=z$
7. $|z|=|\bar{z}|$
8. $|zw|=|z||w|$
9. $|z+w|\leq|z|+|w|$
10. $z^{-1}=\frac{1}{z}=\frac{\bar{z}}{|z|^2}$ when $z\neq 0+0i$

$z=a+bi$ and $w=c+di$.

Imaginary unit number:	$i=\sqrt{-1}$
Complex numbers addition:	$(a+bi)+(c+di)=(a+c)+i(b+d)$
Complex numbers multiplication:	$(a+bi)(c+di)=(ac-bd)+i(ad+bc)$
Complex conjugate:	$\bar{z}=a-bi$
Modulus of a complex number:	$\lvert z\rvert=\sqrt{z\bar{z}}=\sqrt{a^2+b^2}$
Euler's formula:	$e^{i\theta}=\cos\theta+i\sin\theta$
Polar form:	$z=\lvert z\rvert e^{i\theta}$
Periodicity of complex numbers:	$e^{i\theta\pm 2\pi}=e^{i\theta}$

The Complex Number System

- It is the extension of the real number system via closure under exponentiation.

$$i \equiv \sqrt{-1}$$

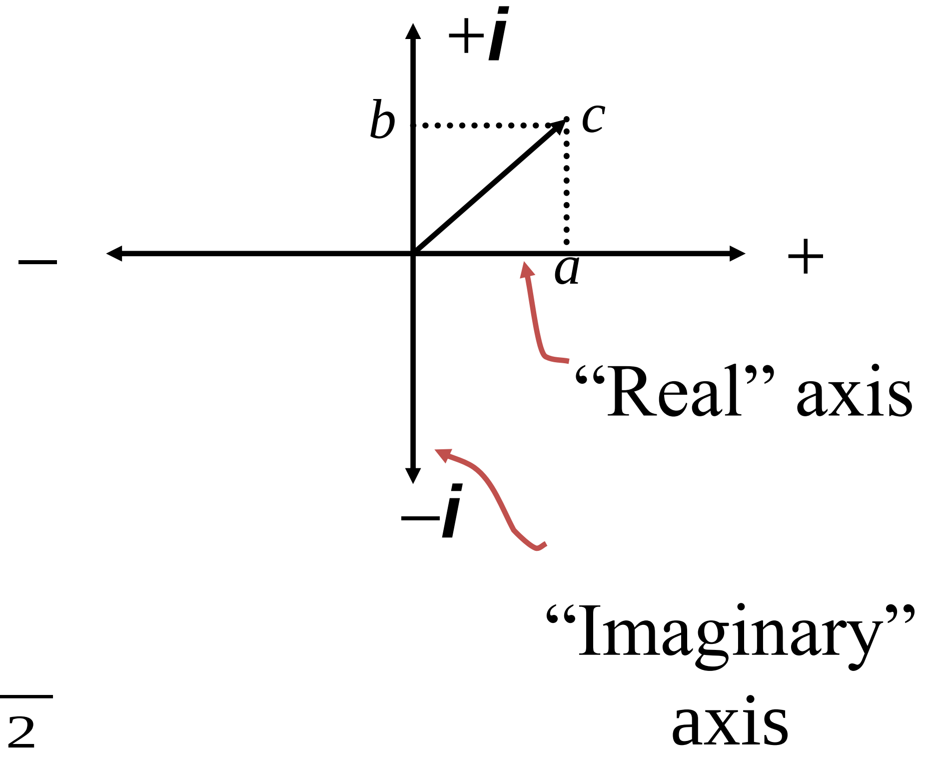
The "imaginary" unit

$$c = a + bi$$

$$\mathcal{Re}[c] \equiv a$$

$$\mathcal{Im}[c] \equiv b$$

$$(c \in \mathbf{C}, a, b \in \mathbf{R})$$



- *(Complex) conjugate:*

$$c^* = (a + bi)^* \equiv (a - bi)$$

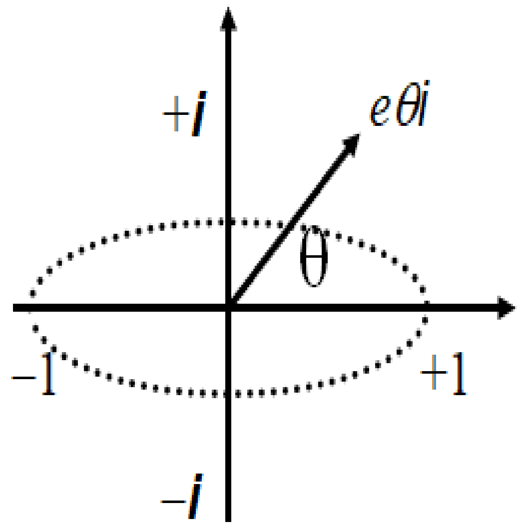
- *Magnitude* or *absolute value*:

$$|c|^2 = c^*c = a^2+b^2$$

$$|c| \equiv \sqrt{c^* c} = \sqrt{(a - bi)(a + bi)} = \sqrt{a^2 + b^2}$$

Complex Exponentiation

- Powers of *i* are complex units:



- Note: $e^{\theta i} \equiv \cos\theta + i\sin\theta$

$e^{\pi i/2} = i$

$e^{\pi i} = -1$

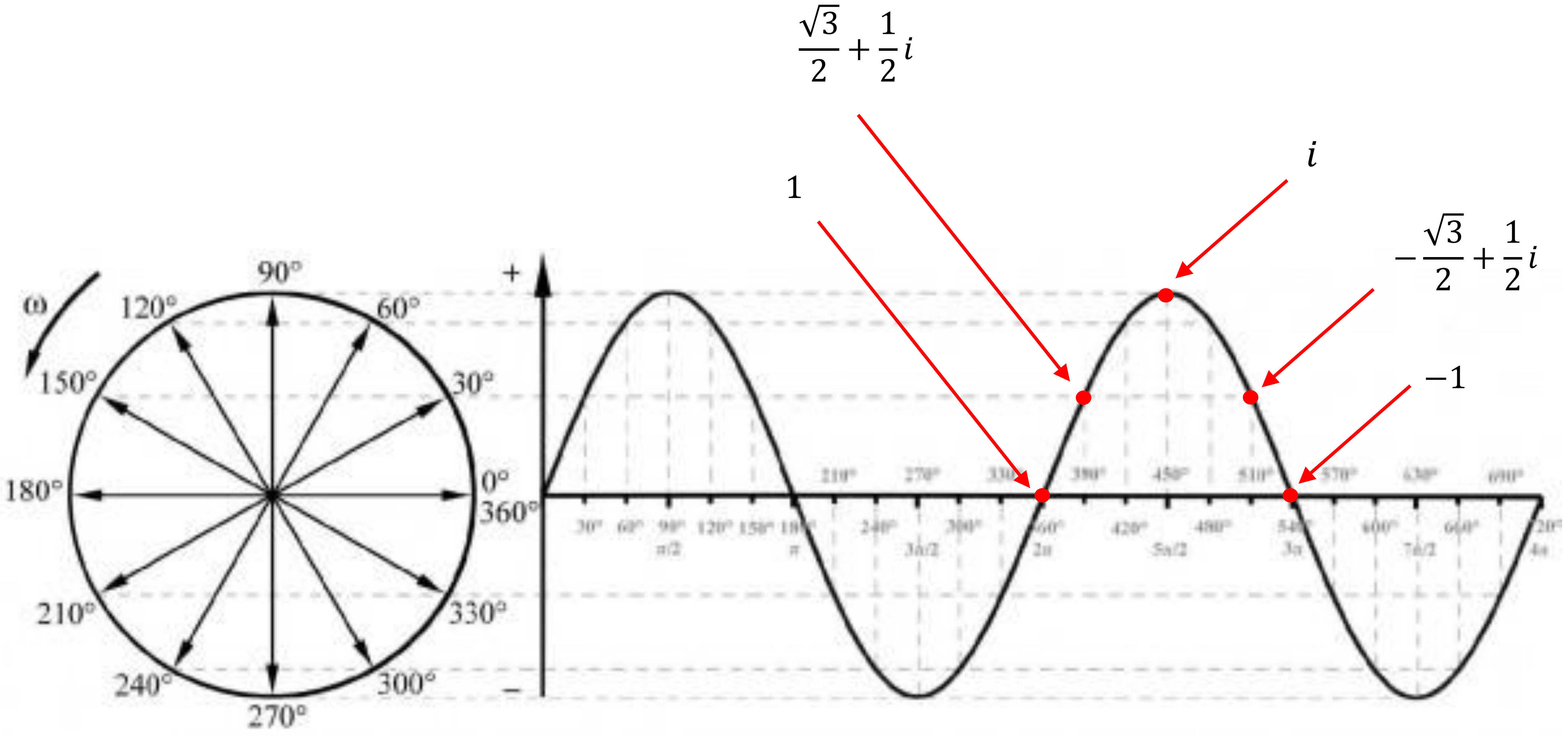
$e^{3\pi i/2} = -i$

$e^{2\pi i} = e^0 = 1$

Z1=2 e^πi

Z12 = (2 e πi)^2 = 2^2 (e πi)^2 = 4 (e πi)^2 = 4 e^2πi

Waves



$\frac{\sqrt{3}}{2}+\frac{1}{2}i$
i
1
$-\frac{\sqrt{3}}{2}+\frac{1}{2}i$
-1
ω
90°
120°
60°
150°
30°
180°
0°
360°
210°
330°
240°
300°
270°
+
–

Review of complex exponential

geometric series is used repeatedly to simplify expressions.

$$\sum_{n=0}^{n-1} x^n = 1 + x + x^2 + \ldots + x^{N-1} = \frac{1-x^n}{1-x}$$

- if the magnitude of x is less than one, then

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \qquad |x| < 1$$

The geometric series is often a complex exponential variable of the form e^{jk}, where j = $\sqrt{-1}$

Complex Numbers

- Euler's formula

$$e^{\pm j\theta} = cos(\theta) \pm jsin(\theta)$$

- Properties

$$|e^{\pm j\theta}| = \sqrt{\cos^2(\theta) + \sin^2(\theta)} = 1$$

$$\phi(e^{\pm j\theta}) = \tan^{-1}(\pm \frac{sin(\theta)}{cos(\theta)}) = \tan^{-1}(\pm tan(\theta)) = \pm\theta$$

$$sin(\theta) = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$$

$$cos(\theta) = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$$

Örnek

What's the polar form of $z = 5 - 5i$? We first need to find the modulus of z, which is given by:

$$|z| = \sqrt{5^2 + (-5)^2}$$
$$= \sqrt{50}$$

The argument is given by:

$$\theta = \arctan\left(\frac{5}{-5}\right)$$
$$= \arctan(-1)$$
$$= \frac{3\pi}{4} \text{ or } \frac{7\pi}{4}$$

Since the the real part of z is positive and its imaginary is negative,

$$\theta = \frac{7\pi}{4}$$

$$5 - 5i = \sqrt{50}e^{i\frac{7\pi}{4}}$$

Properties of the polar form

$z = |z|e^{i\theta}$ and $w = |w|e^{i\phi}$

1. $e^{i\theta}e^{i\phi} = e^{i(\theta+\phi)} \implies zw = (|z|e^{i\theta})(|w|e^{i\phi}) = |z||w|e^{i(\theta+\phi)}$
2. $(e^{i\theta})^n = e^{in\theta}$, for any number n (i.e., n could be complex!)
3. From the above property $\Rightarrow \dfrac{1}{e^{i\theta}} = (e^{i\theta})^{-1} = e^{-i\theta}$
4. $|e^{i\theta}| = e^{i\theta} \cdot \overline{e^{i\theta}} = e^{i\theta}e^{-i\theta} = e^{i(\theta-\theta)} = e^0 = 1$
5. $\overline{e^{i\theta}} = e^{-i\theta}$
6. Since $e^{\pm 2\pi i} = \cos(\pm 2\pi) + i\sin(\pm 2\pi) = 1$, then $e^{i(\theta \pm 2\pi)} = e^{i\theta} \cdot e^{\pm 2\pi i} = e^{i\theta}$

Trigonometric Identities

Trigonometric Identities

$$\sin\theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$$

$$\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\sin(\theta+\alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha$$

$$\cos(\theta+\alpha) = \cos\theta\cos\alpha - \sin\theta\sin\alpha$$

$$\sin\theta\sin\alpha = \frac{1}{2}(\cos(\theta-\alpha) - \cos(\theta+\alpha))$$

$$\cos\theta\cos\alpha = \frac{1}{2}(\cos(\theta+\alpha) + \cos(\theta-\alpha))$$

$$\sin\theta\cos\alpha = \frac{1}{2}(\sin(\theta+\alpha) + \sin(\theta-\alpha))$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Trigonometric Formula

$$\cos^2 A + \sin^2 A = 1 \qquad \sec^2 A - \tan^2 A = 1 \qquad \operatorname{cosec}^2 A - \cot^2 A = 1$$

$$\sin 2A = 2\sin A\cos A \qquad \cos 2A = \cos^2 A - \sin^2 A \qquad \tan 2A = \frac{2\tan A}{1-\tan^2 A}.$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B \qquad \cos A\cos B = \frac{\cos(A+B)+\cos(A-B)}{2}$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B \qquad \sin A\sin B = \frac{\cos(A-B)-\cos(A+B)}{2}$$

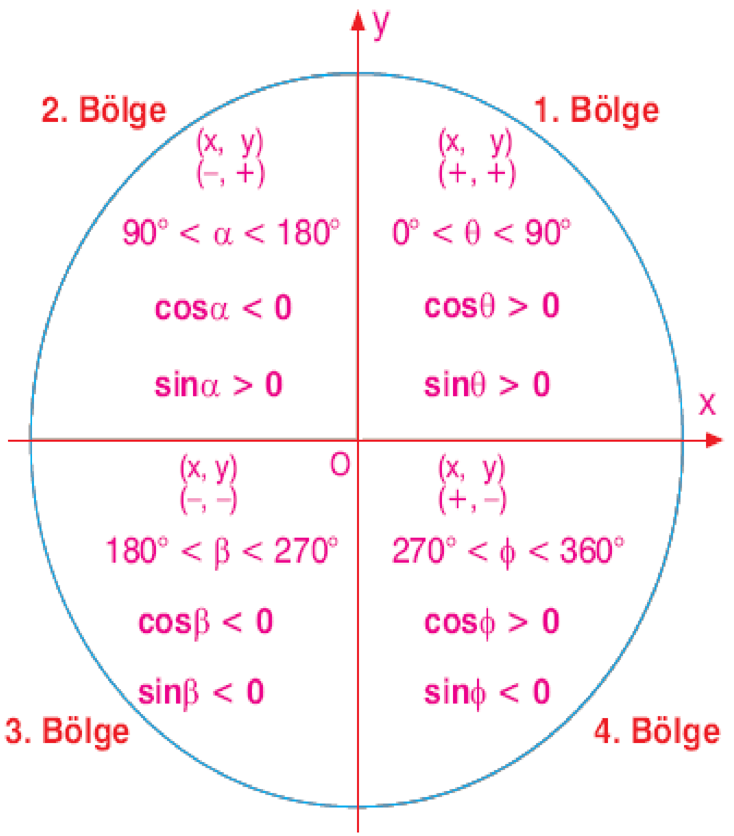
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B} \qquad \sin A\cos B = \frac{\sin(A+B)+\sin(A-B)}{2}$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} \qquad \cos^2 A = \frac{1+\cos 2A}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} \qquad \sin^2 A = \frac{1-\cos 2A}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} \qquad \cos^3 A = \frac{3\cos A+\cos 3A}{4}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2} \qquad \sin^3 A = \frac{3\sin A-\sin 3A}{4}$$



	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Tanımsız

$$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$$

$$\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$$

$$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$$

$$\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$$

$$\sin(\pi-\theta)=\sin\theta$$

$$\sin(\pi+\theta)=-\sin\theta$$

$$\cos(\pi-\theta)=-\cos\theta$$

$$\cos(\pi+\theta)=-\cos\theta$$

Sine and Cosine Addition and Subtraction Formulas

$$\sin(a+b)=\sin a\cos b+\cos a\sin b$$

$$\sin(a-b)=\sin a\cos b-\cos a\sin b$$

$$\cos(a+b)=\cos a\cos b-\sin a\sin b$$

$$\cos(a-b)=\cos a\cos b+\sin a\sin b$$

$$\cos x=\frac{1}{2}\left(e^{ix}+e^{-ix}\right)$$

$$\sin x=\frac{1}{2i}\left(e^{ix}-e^{-ix}\right)$$

Exponential Function

Exponential Function

- The function defined by is called an exponential function with base *b* and exponent *x*.
- The domain of *f* is the set of all real numbers.

$$f(x) = b^x \qquad (b > 0,\ b \neq 1)$$

- The exponential function with base 2 is the function with domain $(-\infty, \infty)$.
- The values of $f(x)$ for selected values of x follow:

$$f(x) = 2^x \qquad f(3) = 2^3 = 8$$

$$f(0) = 2^0 = 1$$

Laws of Exponents

- Let *a* and *b* be positive numbers and let *x* and *y* be real numbers. Then,

1. $b^x \cdot b^y = b^{x+y}$
2. $\dfrac{b^x}{b^y} = b^{x-y}$
3. $\left(b^x\right)^y = b^{xy}$
4. $(ab)^x = a^x b^x$
5. $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

Examples

- Sketch the graph of the exponential function $f(x) = 2^x$.

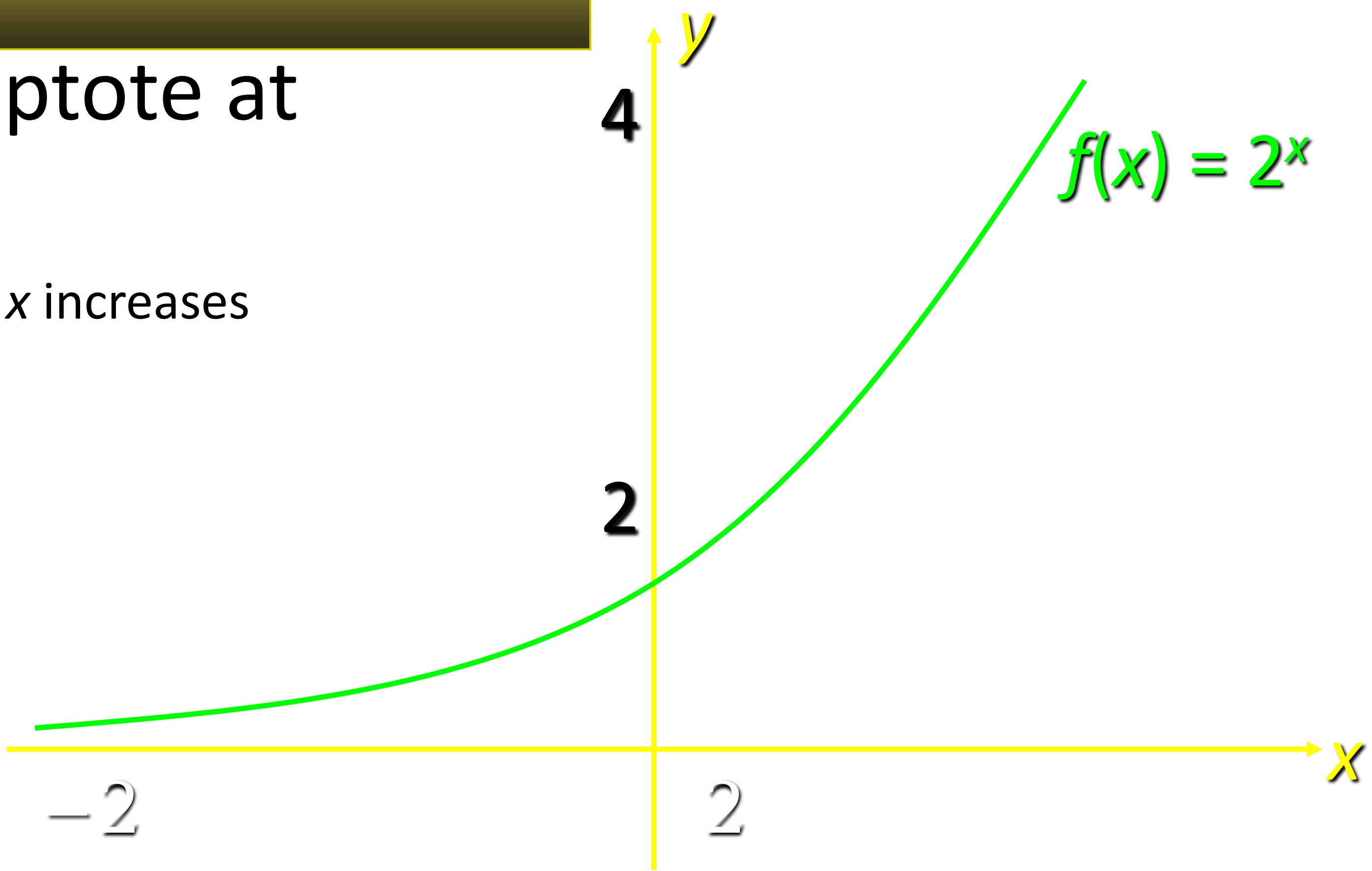
<u>Solution</u>

- Now, consider a few values for x:

x	– 5	– 4	– 3	– 2	– 1	0	1	2	3	4	5
y	1/32	1/16	1/8	1/4	1/2	1	2	4	8	16	32

-
  – There is a horizontal asymptote at $y = 0$.

- Furthermore, 2^x increases without bound when x increases without bound.
- Thus, the range of f is the interval $(0, \infty)$.



Properties of Exponential Functions

- The exponential function $y = b^x$ ($b > 0$, $b \neq 1$) has the following properties:

1. Its domain is $(-\infty, \infty)$.
2. Its range is $(0, \infty)$.
3. Its graph passes through the point (0, 1)
4. It is continuous on $(-\infty, \infty)$.
5. It is increasing on $(-\infty, \infty)$ if $b > 1$ and decreasing on $(-\infty, \infty)$ if $b < 1$.

The Base *e*

- Exponential functions to the base *e*, where *e* is an irrational number whose value is 2.7182818…, play an important role in both theoretical and applied problems.
- It can be shown that

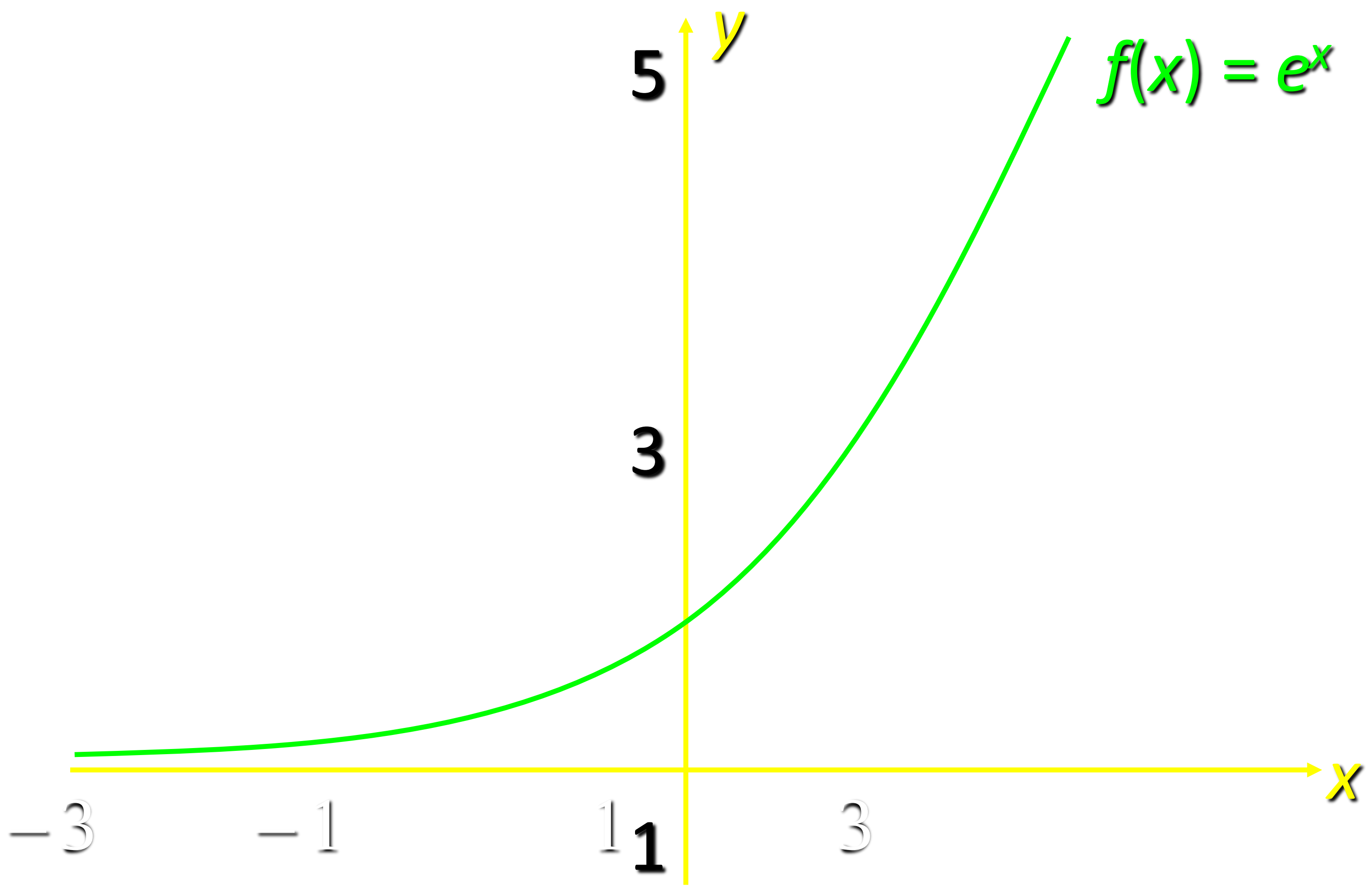
$$e = \lim_{m \to \infty} \left(1 + \frac{1}{m}\right)^m$$

Examples

- Sketch the graph of the exponential function $f(x) = e^x$.

Solution

- Sketching the graph:



Examples

- Sketch the graph of the exponential function $f(x) = e^{-x}$.

Solution

- Since $e^{-x} > 0$ it follows that $0 < 1/e < 1$ and so

  $f(x) = e^{-x} = 1/e^x = (1/e)^x$ is an exponential function with base less than 1.
- Therefore, it has a graph similar to that of $y = (1/2)^x$.
- Consider a few values for x:

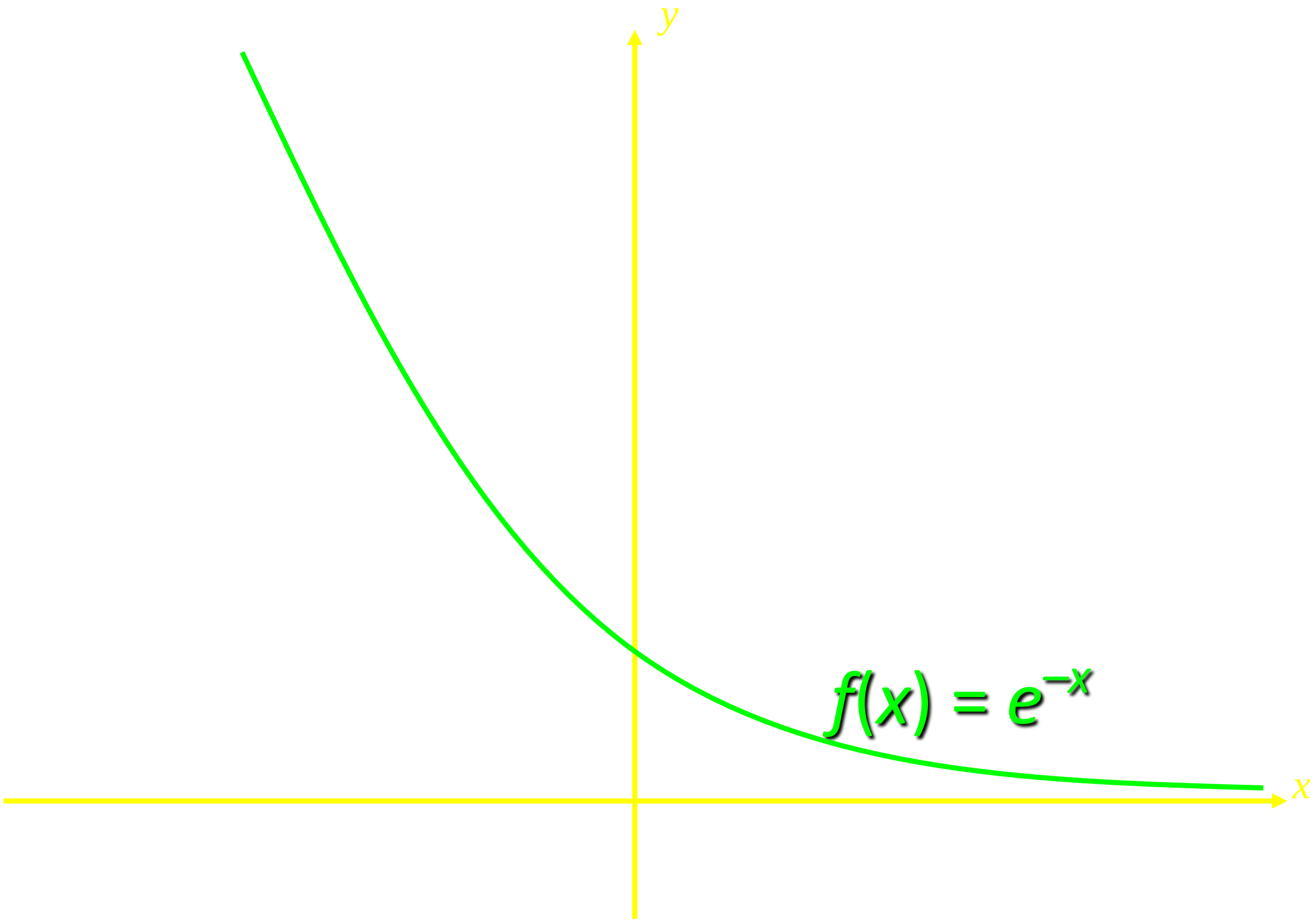
x	– 3	– 2	– 1	0	1	2	3
y	20.09	7.39	2.72	1	0.37	0.14	0.05

Examples

- Sketch the graph of the exponential function $f(x) = e^{-x}$.

<u>Solution</u>

- Sketching the graph:



Türev

1. $\frac{d}{dx}(c) = 0,$ where c is a constant

2. $\frac{d}{dx}(x^n) = nx^{n-1},$ where n is any real number

3. $\frac{d}{dx}(e^x) = e^x,$ $\frac{d}{dx}(e^{cx}) = ce^{cx}$

4. $\frac{d}{dx}(\ln x) = \frac{1}{x},$ for $x>0$

5. $\frac{d}{dx}(\sin x) = \cos x$

6. $\frac{d}{dx}(\cos x) = -\sin x$

Differentiation

$$(uv)' = u'v + uv', \quad \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$(uv)^{(n)} = u^{(n)}v + nu^{(n-1)}v^{(1)} + \cdots + {}^nC_r u^{(n-r)}v^{(r)} + \cdots + uv^{(n)}$$

$$\text{where } {}^nC_r \equiv \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}(\sin x) &= \cos x & \frac{\mathrm{d}}{\mathrm{d}x}(\sinh x) &= \cosh x \\
\frac{\mathrm{d}}{\mathrm{d}x}(\cos x) &= -\sin x & \frac{\mathrm{d}}{\mathrm{d}x}(\cosh x) &= \sinh x \\
\frac{\mathrm{d}}{\mathrm{d}x}(\tan x) &= \sec^2 x & \frac{\mathrm{d}}{\mathrm{d}x}(\tanh x) &= \operatorname{sech}^2 x \\
\frac{\mathrm{d}}{\mathrm{d}x}(\sec x) &= \sec x \tan x & \frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{sech} x) &= -\operatorname{sech} x \tanh x \\
\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) &= -\operatorname{cosec}^2 x & \frac{\mathrm{d}}{\mathrm{d}x}(\coth x) &= -\operatorname{cosech}^2 x \\
\frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{cosec} x) &= -\operatorname{cosec} x \cot x & \frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{cosech} x) &= -\operatorname{cosech} x \coth x
\end{aligned}$$

Integral

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \frac{1}{x} \, dx = \ln x + c$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + c$$

$$\int \ln x \, dx = x(\ln x - 1) + c$$

$$\int x e^{ax} \, dx = e^{ax} \left(\frac{x}{a} - \frac{1}{a^2} \right) + c$$

$$\int_a^b u \, dv = uv \Big|_a^b - \int_a^b v \, du$$

$$(uv)' = u'v + uv', \quad \left(\frac{u}{v} \right)' = \frac{u'v - uv'}{v^2}$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int e^x dx = e^x$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int b^{ax} dx = \frac{1}{a \ln(b)} b^{ax} \qquad ; b > 0$$

$$\int \ln(x) \, dx = x \ln(x) - x$$

$$\int a^x \ln(a) dx = a^x \qquad ; a > 0$$

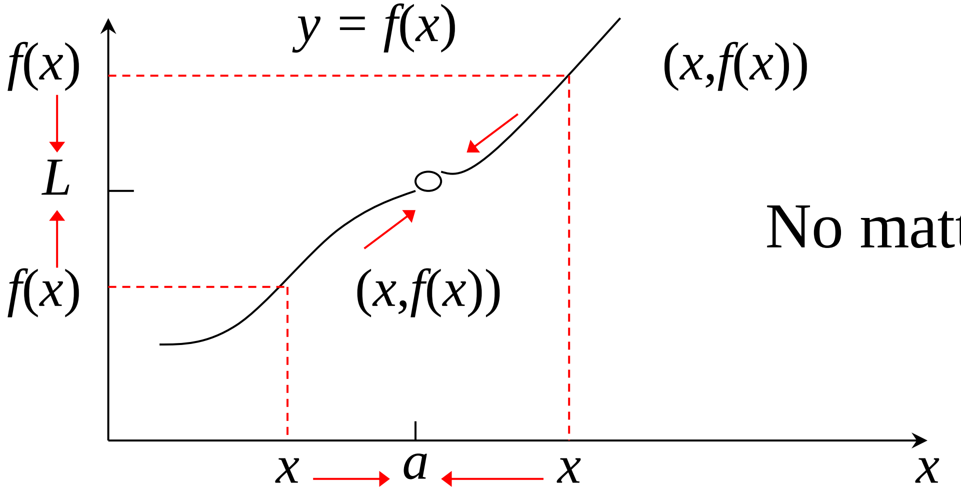
1. $\frac{d}{dx}(c) = 0,$ where c is a constant
2. $\frac{d}{dx}(x^n) = nx^{n-1},$ where n is any real number
3. $\frac{d}{dx}(e^x) = e^x$
4. $\frac{d}{dx}(\ln x) = \frac{1}{x},$ for $x>0$
5. $\frac{d}{dx}(\sin x) = \cos x$
6. $\frac{d}{dx}(\cos x) = -\sin x$

Limit

Definition of Limit of a Function

Suppose that the function $f(x)$ is defined for all values of x near a, but not necessarily at a. If as x approaches a (without actually attaining the value a), $f(x)$ approaches the number L, then we say that L is the limit of $f(x)$ as x approaches a, and write

$$\lim_{x \to a} f(x) = L$$



No matter how x approaches a, $f(x)$ approaches L.

Properties of Limits and Direct Substitution

By combining the basic limits with the following operations, you can find limits for a wide variety of functions.

Properties of Limits

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = K$$

1. Scalar multiple: $\lim_{x \to c} [bf(x)] = bL$

2. Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$

3. Product: $\lim_{x \to c} [f(x)g(x)] = LK$

4. Quotient: $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}, \quad K \neq 0$

5. Power: $\lim_{x \to c} [f(x)]^n = L^n$

Properties of Limits and Direct Substitution

The following summarizes the results of using direct substitution to evaluate limits of polynomial and rational functions.

Limits of Polynomial and Rational Functions

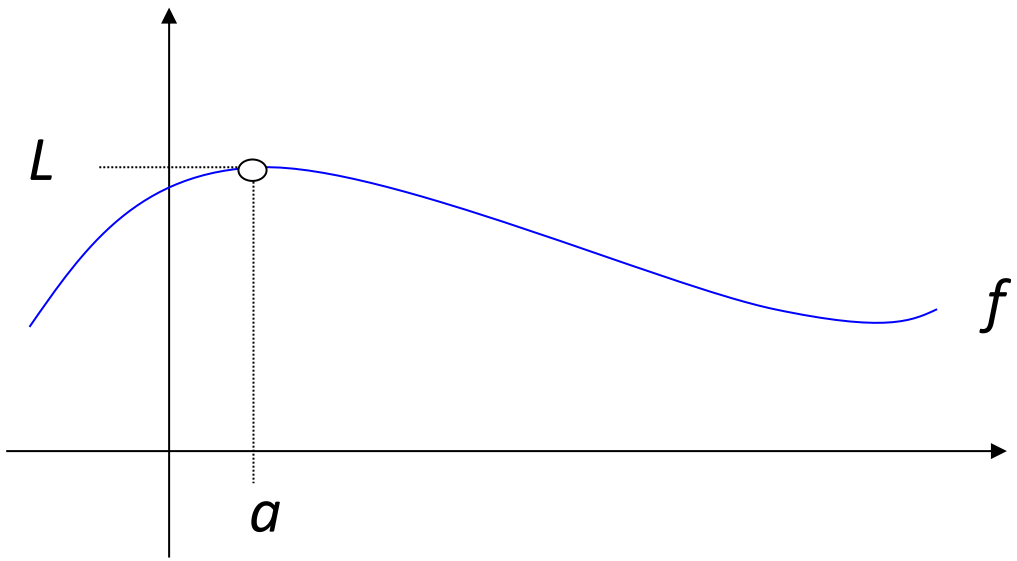
1. If p is a polynomial function and c is a real number, then

$$\lim_{x \to c} p(x) = p(c).$$

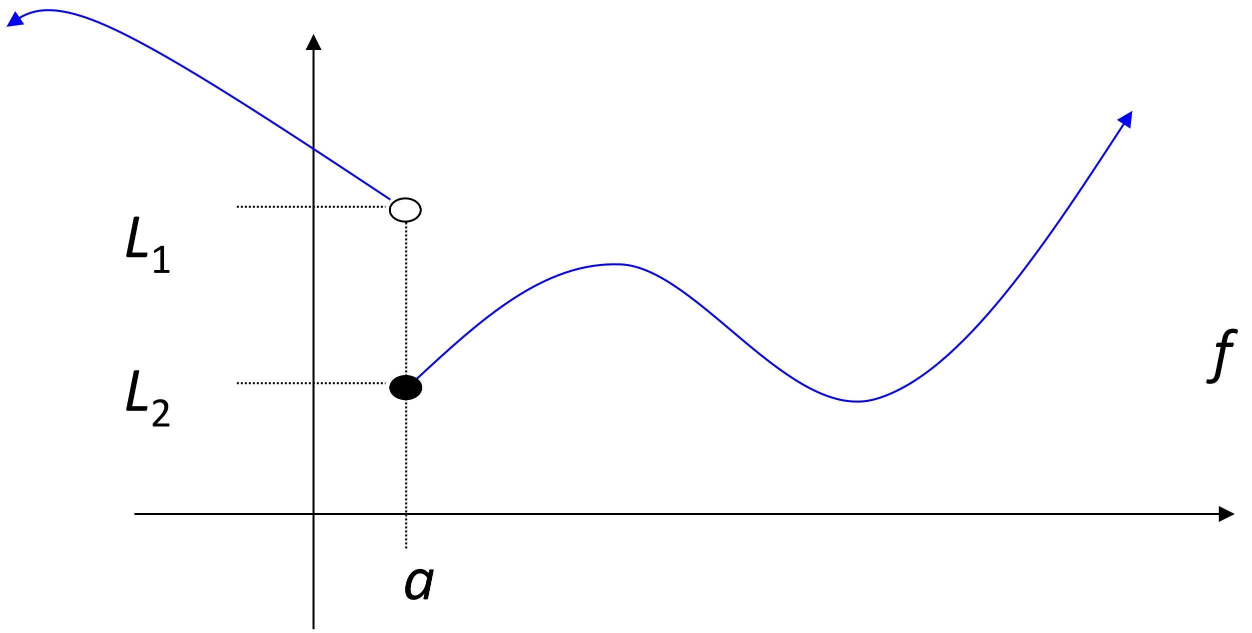
2. If r is a rational function $r(x) = p(x)/q(x)$, and c is a real number such that $q(c) \neq 0$, then

$$\lim_{x \to c} r(x) = r(c) = \frac{p(c)}{q(c)}.$$

Possible Limit Situations



$$\lim_{x \to a} f(x) = L$$

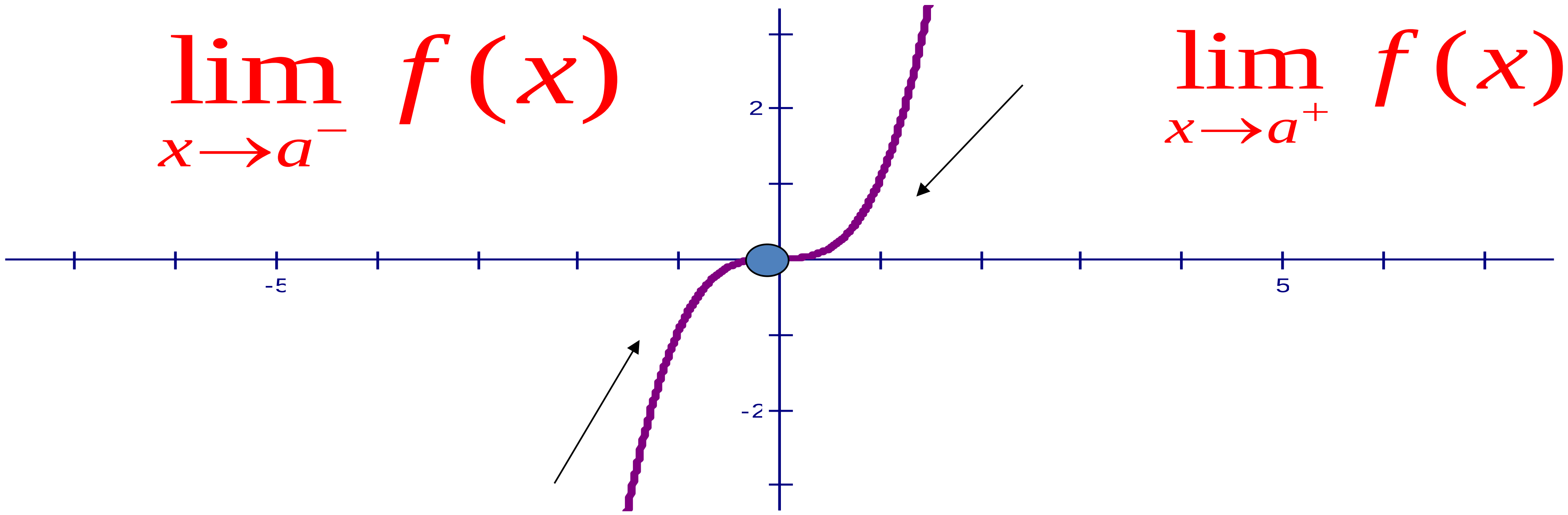


$$\lim_{x \to a} f(x) = DNE$$

DNE = Does Not Exist

Left & Right Hand Limits

$$\lim_{x \to a^-} f(x)$$



$$\lim_{x \to a^+} f(x)$$

Definition: One Sided Limits

Left-Hand Limit: The limit of *f* as *x* approaches *a* from the left equals *L* is denoted

$$\lim_{x \to a^-} f(x) = L$$

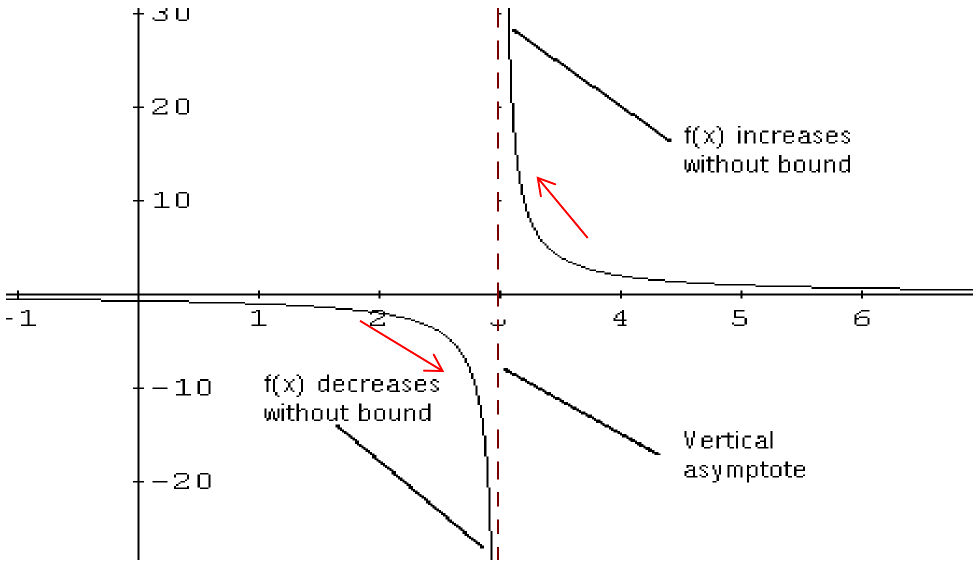
Right-Hand Limit: The limit of *f* as *x* approaches *a* from the right equals *L* is denoted

$$\lim_{x \to a^+} f(x) = L$$

Evaluating Limits Graphically
Limits that do not exist

f(x) increases or decreases without bound as *x* approaches *c*.

6. $f(x) = \dfrac{2}{x-3}$



$$\lim_{x \to 3} f(x) = dne$$

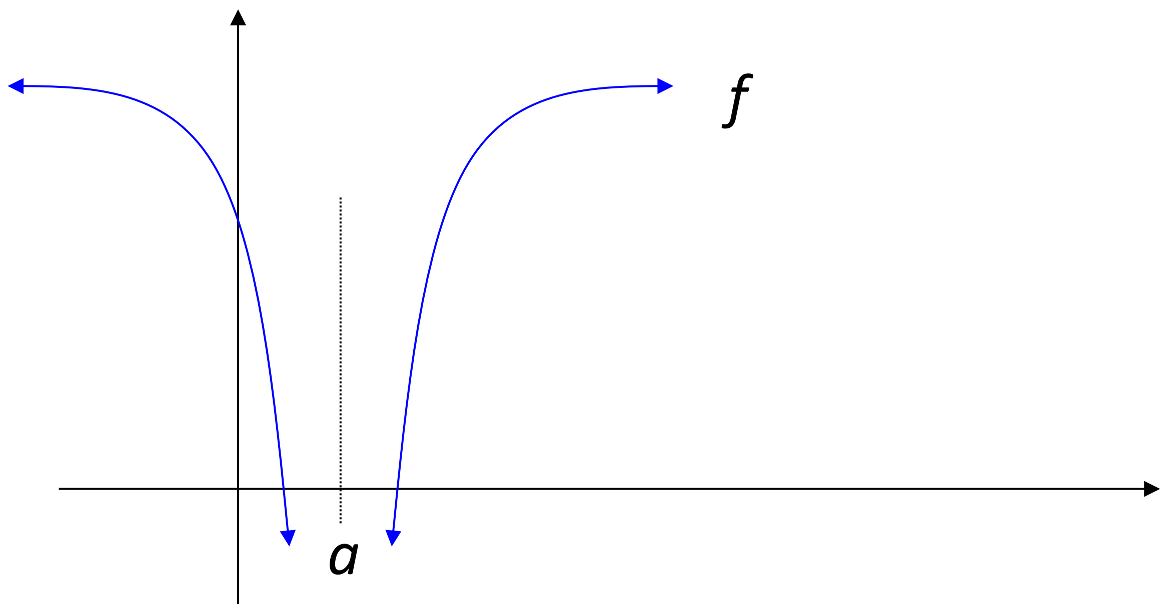
This function is undefined at x = 3, because the denominator goes to zero. It can not be simplified, so there is a vertical asymptote at x = 3.

Approaching 3 from the right, *f(x)* increases without bound.

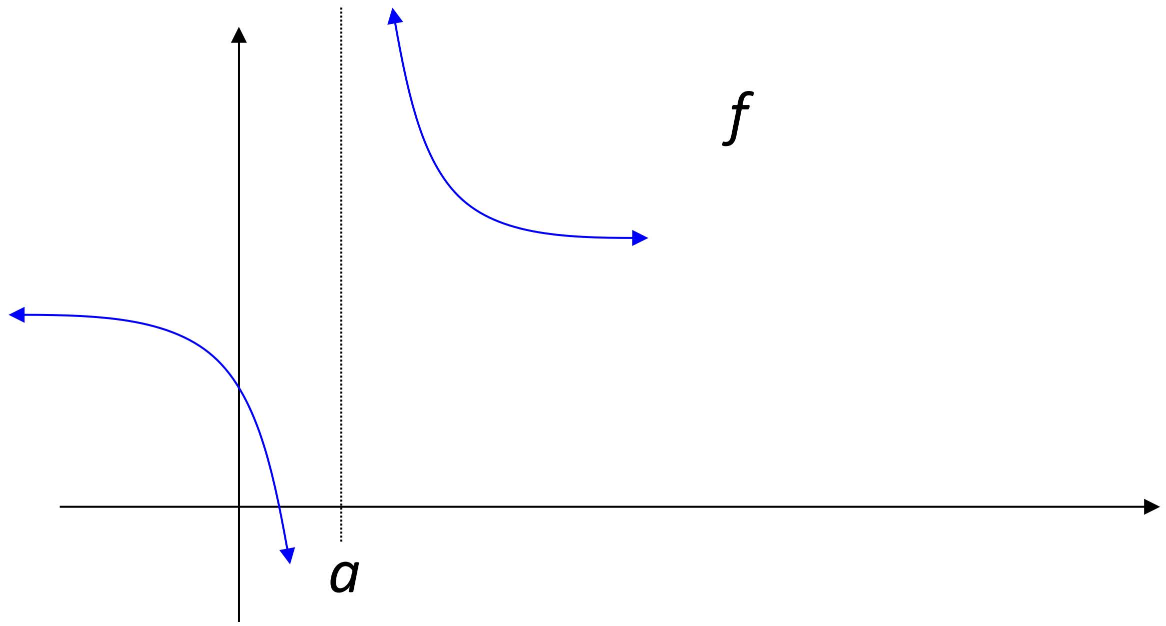
Approaching 3 from the left, *f(x)* decreases without bound.

When the function increases or decreases without bound, the limit does not exist.

Possible Limit Situations



$$\lim_{x \to a} f(x) = -\infty$$



$$\lim_{x \to a} f(x) = DNE$$

DNE = Does Not Exist

Example – *Dividing Out Technique*

- Find the limit.

$$\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3}$$

$$\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3} = \lim_{x \to -3} \frac{(x - 2)(x + 3)}{x + 3}$$

THE SUM LAW

The limit of a sum is the sum of the limits.

$$\lim_{x \to a}[f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

THE DIFFERENCE LAW

The limit of a difference is the difference of the limits.

$$\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$$

THE CONSTANT MULTIPLE LAW

The limit of a constant times a function is the constant times the limit of the function.

$$\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$$

THE PRODUCT LAW

The limit of a product is the product of the limits.

$$\lim_{x \to a}[f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

THE QUOTIENT LAW

The limit of a quotient is the quotient of the limits (provided that the limit of the denominator is not 0).

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \quad \text{if } \lim_{x \to a} g(x) \neq 0$$

THE POWER LAW

If we use the Product Law repeatedly with $f(x) = g(x)$, we obtain the Power Law.

$$6. \lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$$

where n is a positive integer

In applying these six limit laws, we need to use two special limits.

$$7. \lim_{x \to a} c = c$$

$$8. \lim_{x \to a} x = a$$

- These limits are obvious from an intuitive point of view.
- State them in words or draw graphs of $y = c$ and $y = x$.

If we now put $f(x) = x$ in the Power Law and use Law 8, we get another useful special limit.

$$9. \lim_{x \to a} x^n = a^n$$

where n is a positive integer.

Finding a Limit at Infinity

$$\lim_{x\to\infty}\frac{5x^2-7x+1}{2x^2+x+5}=\lim_{x\to\infty}\frac{5-\frac{7}{x}+\frac{1}{x^2}}{2+\frac{1}{x}+\frac{5}{x^2}}$$

$$\lim_{x\to\infty}\frac{5x^2-7x+1}{2x^2+x+5}=\frac{\lim_{x\to\infty}5-7\cdot\lim_{x\to\infty}\frac{1}{x}+\lim_{x\to\infty}\frac{1}{x^2}}{\lim_{x\to\infty}2+\lim_{x\to\infty}\frac{1}{x}+5\cdot\lim_{x\to\infty}\frac{1}{x^2}}$$

$$=\frac{\lim_{x\to\infty}5-\frac{7}{x}+\frac{1}{x^2}}{\lim_{x\to\infty}2+\frac{1}{x}+\frac{5}{x^2}}$$

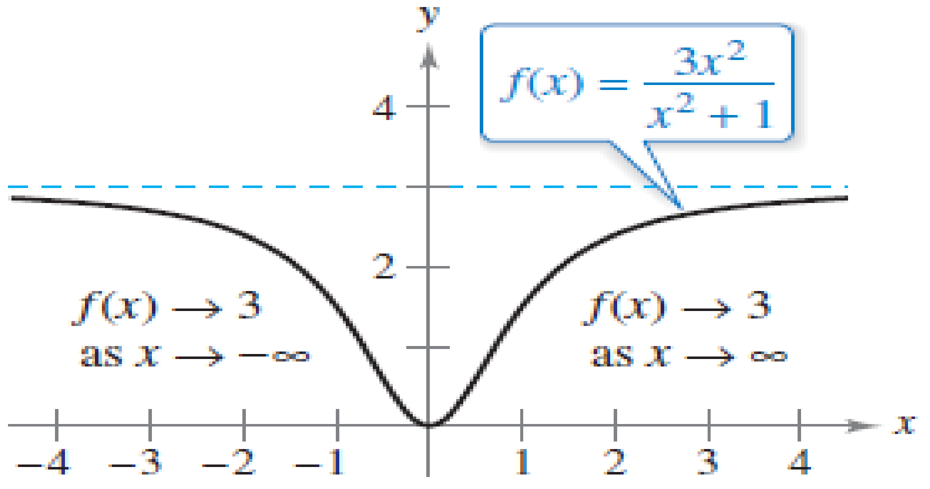
$$=\frac{5-0+0}{2+0+0}=\frac{5}{2}$$

$$\lim_{x\to\infty}\left(5-\frac{2}{x^2}\right)=\lim_{x\to\infty}5-\lim_{x\to\infty}\frac{2}{x^2}=5-0=5.$$

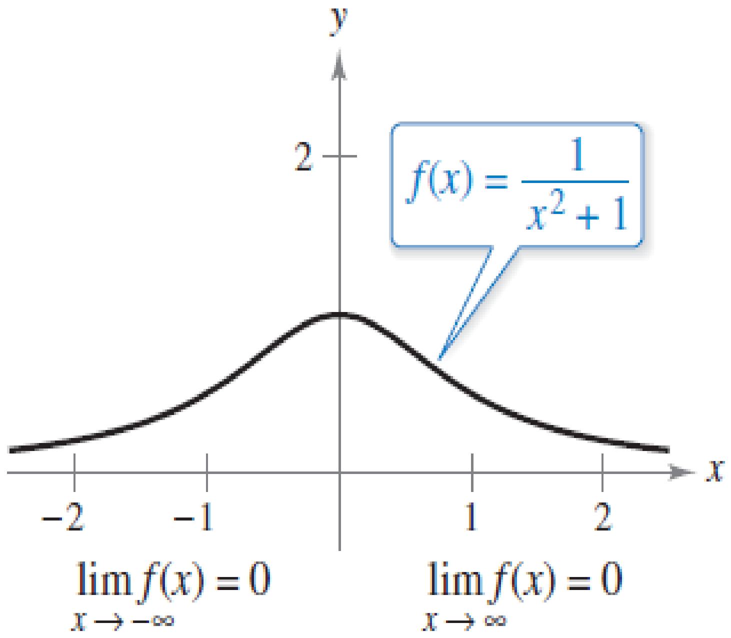
Limits at Infinity

$$f(x) = \frac{3x^2}{x^2 + 1}$$

$$\lim_{x \to -\infty} f(x) = 3 \qquad \lim_{x \to \infty} f(x) = 3.$$



The limit of $f(x)$ as x approaches $-\infty$ or ∞ is 3.



f has a horizontal asymptote at $y = 0$.

Logarithms

Logarithms

- Exponential equations of the form

$$y = b^x \qquad (b > 0,\ b \neq 1)$$

- The logarithm of *x* to the base *b*, and is denoted $\log_b x$.
- Logarithm of *x* to the base *b*

$$y = \log_b x \quad \text{if and only if} \quad x = b^y \qquad (x > 0)$$

$$\log x = \log_{10} x$$ Common logarithm

$$\ln x = \log_e x$$ Natural logarithm

$$y = \log_b x \quad \text{ise} \quad x = b^y$$

Laws of Logarithms

- If *m* and *n* are positive numbers, then

$$\log_b mn = \log_b m + \log_b n$$

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

$$\log_b m^n = n \log_b m$$

$$\log_b 1 = 0$$

$$\log_b b = 1$$

Log1=0, Log 2 ≈ 0.3, Log 3 ≈ 0.5, Log 5 ≈ 0.7, Log 7 ≈ 0.8, Log10=1

Logarithms

- Log1=0, Log 2 ≈ 0.3, Log 3 ≈ 0.477, Log 5 ≈ 0.7, Log 7 ≈ 0.845, Log10=1
- log(a*b)=loga + logb; $loga^n$=n*loga
- 10Log(420)=**10Log(10x7x2x3)=10Log(10) + 10Log(7) + 10Log(3) + 10Log(2) = 10 + 8 + 5 + 3 = 26**
- 10Log(75)=**10Log($3*5^2$)= 10Log(3) + 10Log(5^2) = 5 + 20log(5) =5+14=19**
- P_{dBW}=10log(P_W);
- P_{dBm}=10log(Pm_W); 1W=10^3 mW.   1mW=10^{-3} Watt
- K_{dB}=10log(Po/Pi); Po:çıkış gücü(w), Pi: giriş gücü(W).
- Bir sistemin güç çıkışı 1 watt'tır. Giriş gücü 8 watt. Güç kazancını logaritmik değer olarak hesaplayınız? Bu güç kazancı kazanç mı yoksa kayıp mı?
  - **K=10log(1/8)=10Log(1)-10Log(2^3)=0-30Log2=-9dB. Kayıptır çünkü, K<0.**

Properties of Logarithmic Functions

- The logarithmic function $y = \log_b x$ ($b > 0$, $b \neq 1$) has the following properties:
  1. Its domain is $(0, \infty)$.
  2. Its range is $(-\infty, \infty)$.
  3. Its graph passes through the point $(1, 0)$.
  4. It is continuous on $(0, \infty)$.
  5. It is increasing on $(0, \infty)$ if $b > 1$ and decreasing on $(0, \infty)$ if $b < 1$.

Exponential Logarithmic Functions

- Solve the equation $2e^{x+2} = 5$.

<u>Solution</u>

- Divide both sides of the equation by 2 to obtain:

$$e^{x+2} = \frac{5}{2} = 2.5$$

- Take the natural logarithm of each side of the equation and solve:

$$\ln e^{x+2} = \ln 2.5$$
$$(x+2)\ln e = \ln 2.5$$
$$x + 2 = \ln 2.5$$
$$x = -2 + \ln 2.5$$
$$x \approx -1.08$$

- Properties relating e^x and $\ln x$: $e^{\ln x} = x$ $(x > 0)$

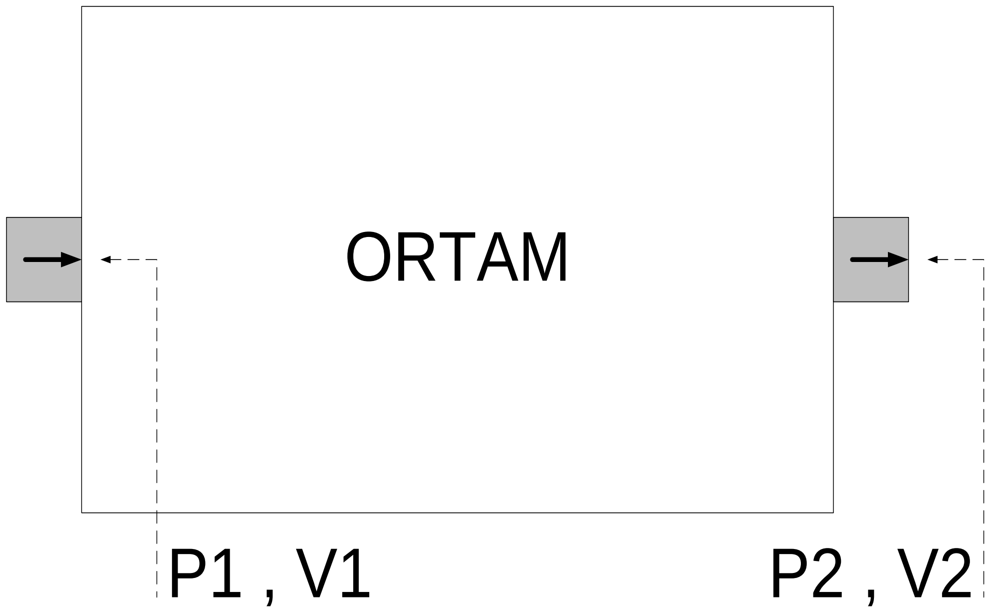
  $\ln e^x = x$ (for any real number x)

Kazanç – Kayıp ve Desibel Tanımları

dB, dBm, dBw

1) dB- desibel iki güç seviyesi oranı ilen tanımlandığından birimsiz sayıdır.

$$dB \equiv 10 Log_{10}(\frac{P_2}{P_1})$$



İki güç seviyesi birbirleri ile orantı temelinde ilişkilidir. Eğer P2 güç seviyesi P1 güç seviyesinden büyük ise dB pozitiftir. Tersi durumda negatiftir. $P = \frac{V^2}{R}$, eşit veya aynı direnç değerlerinde gerilimler ölçüldüğünde dB değeri gerilimler cinsinden aşağıdaki biçimde yazılır.

$$dB \equiv 20 Log_{10}(\frac{V_2}{V_1})$$

2) dBW- ölçülen P [Watt] gücünün, 1W referans gücüne oranının logaritmik değeridir.

3) dBm- ölçülen P [Watt] gücünün, 1mW=10^{-3} Watt referans gücüne oranının logaritmik değeridir.

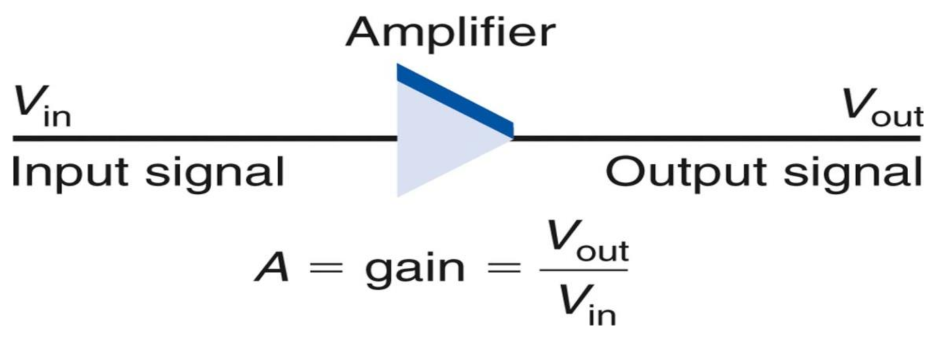
dBm=dBW+30
dBW=dBm-30

Gain, Attenuation, and Decibels

Çıkış işaret seviyesinin giriş işaret seviyesine oranı kazanç, kayıp ya da buffer olarak kendini ifade eder.

Gain

– **Gain** means amplification. It is the ratio of a circuit's output to its input.



An amplifier has gain.

Decibels: Decibel Calculations

- Voltage Gain or Attenuation

  $dB = 20 \log V_{out} / V_{in}$

- Current Gain or Attenuation

  $dB = 20 \log I_{out} / I_{in}$

- Power Gain or Attenuation

  $dB = 10 \log P_{out} / P_{in}$

Power: Gain, Attenuation and Decibels

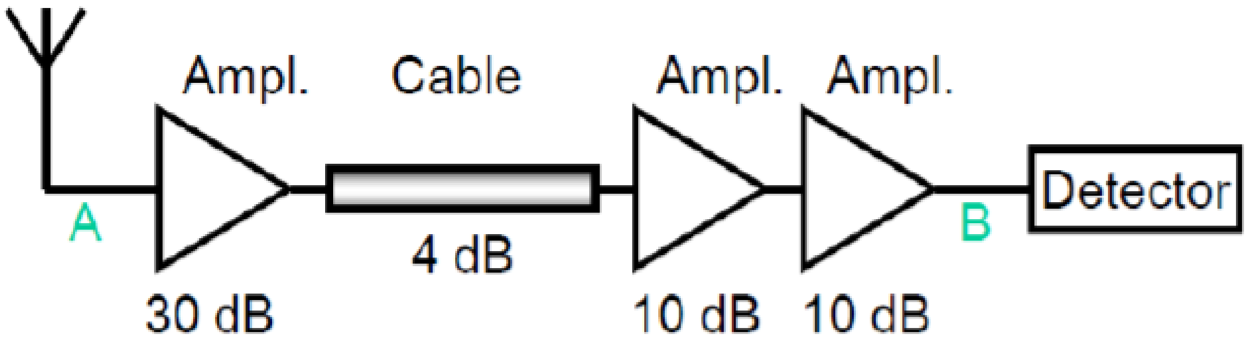
- Most amplifiers are also power amplifiers, so, can be used to calculate power gain K where P_{in} is the power input and P_{out} is the power output.

$$\text{Power gain } (K) = P_{out} / P_{in}$$

- **Example:** The power output of an amplifier is 6 watts (W). The power gain is 80. What is the input power?

$$K = P_{out} / P_{in} \quad \text{therefore} \quad P_{in} = P_{out} / K$$
$$P_{in} = 6 / 80 = 0.075 \text{ W} = 75 \text{ mW}$$

Amplification and Attenuation



The total amplification of the (simplified) receiver chain (between A and B) is

$$G_{A,B}\,|_{dB} = 30 - 4 + 10 + 10 = 46$$

Gain, Attenuation and Decibels

Decibels: Decibel Calculations

- **Example**:

  An amplifier has an input of 3 mV and an output of 5 V. What is the gain in decibels?

  dB = 20 log 5/0.003
  = 20 log 1666.67
  = 20 (3.22)
  = 64.4

Tx Power

Tx is short for "Transmit"

Tx power, the output of a wireless system generates at the RF interface. This power is calculated as the amount of energy given across a defined bandwidth and is usually measured in one of two units:

1. dBm – a relative power level referencing 1 milliwatt
2. dBw – a linear power level referencing Watt

$$dBm = 10 \times \log[PmW]$$

$$dBw = 10 \times \log[Pw]$$

Bir sistemde bir adet dBm (mW) ya da dBw (W) vardır; çok sayıda + ve – lerden oluşan dB ler bulunur.

dBm=dBw+30
dBw=dBm-30

Gain, Attenuation and Decibels

Decibels: Decibel Calculations

- **Example:**

  A filter has a power input of 50 mW and an output of 2 mW. What is the gain or attenuation?

  $$dB = 10 \log (2/50)$$
  $$= 10 \log (0.04)$$
  $$= 10 (-1.398)$$
  $$= -13.98$$

  - If the decibel figure is positive, that denotes a gain.
  - If the decibel figure is positive, that denotes an attenuation.

Matematiksel Fonksiyonlar

Matematiksel Modeller

- Bir matematiksel model hiçbir zaman fiziksel bir durumun tam olarak doğru bir temsili değildir - bir idealleştirmedir.
  - İyi bir model, gerçekliği matematiksel hesaplamalara izin verecek kadar basitleştirir, ancak değerli sonuçlar elde edilecek kadar doğrudur.
  - Modelin sınırlamalarının farkına varmak önemlidir.
  - Sonunda, Doğa Kanunlarının da son sözü vardır, unutlmaz.
- Gerçek dünyada gözlemlenen ilişkileri modellemek için kullanılabilecek birçok farklı işlev türü vardır.

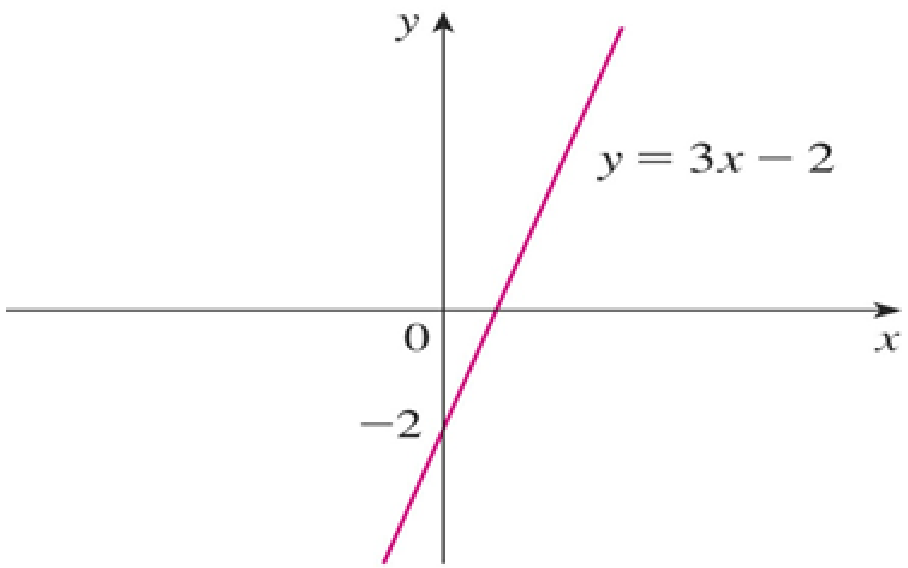
Doğrusal Modeller

- y'nin x'in doğrusal bir fonksiyonu olduğunu söylediğimizde, fonksiyonun grafiğinin bir doğru olduğunu kastediyoruz.
- Böylece, bir doğrunun denkleminin eğim-kesme noktası formunu aşağıdaki fonksiyon için bir formül yazmak için kullanabiliriz:

$$y = f(x) = mx + b$$

burada m, doğrunun eğimi ve b, y kesme noktasıdır.

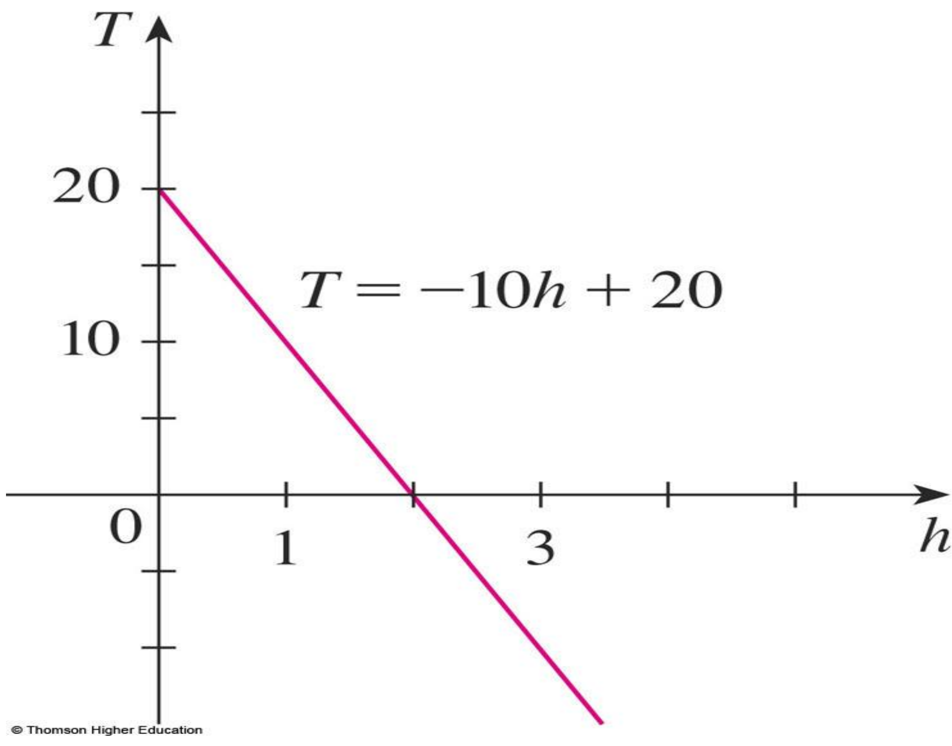
- Doğrusal fonksiyonların karakteristik bir özelliği, sabit bir oranda büyümeleridir.
- Örneğin, şekilde, f (x) = 3x - 2 doğrusal fonksiyonunun bir grafiği ve örnek değerler tablosu verilmiştir.
- 3 değeri grafiğinin eğimi, y'nin x'e göre değişim oranı olarak yorumlanabilir.
- X değeri 0.1 arttığında, f (x) değerinin 0.3 arttığına dikkat edin.
- Yani, f (x), x'in üç katı hızlı artar.



x	$f(x) = 3x - 2$
1.0	1.0
1.1	1.3
1.2	1.6
1.3	1.9
1.4	2.2
1.5	2.5

Doğrusal Modeller

Kuru hava yukarı doğru hareket ettikçe genişler ve soğur. Zemin sıcaklığı 20 ° C ve 1 km yükseklikteki sıcaklık 10 ° C ise, doğrusal bir modelin uygun olduğunu varsayarak sıcaklığı T (° C cinsinden) yüksekliğin (kilometre cinsinden) bir fonksiyonu olarak ifade edin. Fonksiyonun grafiğini çizin. Eğim neyi temsil ediyor? 2.5 km yükseklikte sıcaklık nedir?



T, h'nin doğrusal bir fonksiyonu olduğunu varsaydığımız için, T = mh + b yazabiliriz.

h = 0, yani 20 = m*0+b olduğunda, y kesme noktası b = 20'dir. Ayrıca, h = 1 olduğunda T = 10, m=-10 olur.

Gerekli doğrusal fonksiyon T = -10h + 20'dir.

Eğim m = -10 ° C / km'dir.

Bu, yüksekliğe göre sıcaklık değişim oranını temsil eder.

h = 2,5 km yükseklikte sıcaklık: T = -10 (2,5) + 20 = -5 ° C'dir.

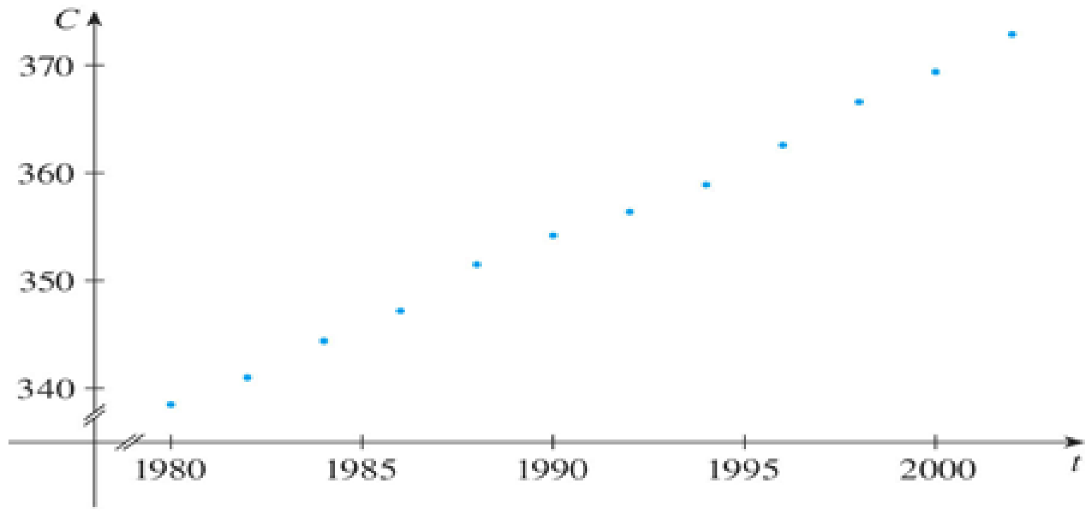
Deneysel Model (Empirical Model)

Bir modeli formüle etmemize yardımcı olacak fiziksel bir yasa veya ilke ya da matematiksel bir denklem yoksa, deneysel bir model oluşturulur.

Deneysel Model tamamen toplanan verilere dayanmaktadır. Veri noktalarının temel eğilimini yakalaması anlamında verilere "uyan" bir eğri aranır.

Örnek: Tablo, 1980'den 2002'ye kadar Mauna Loa Gözlemevi'nde milyonda parça olarak ölçülen atmosferdeki ortalama karbondioksit (CO2) seviyesini listelemektedir. CO2 seviyesi için bir model bulmak için verileri kullanın. Lineer model oluşturun.

Şekilde gösterilen dağılım grafiğini yapmak için tablodaki verileri kullanırız. Grafikte t zamanı (yıl olarak) ve C, CO2 seviyesini (milyonda parça, ppm) temsil eder.



Scatter plot for the average CO_2 level

Year	CO_2 level (in ppm)	Year	CO_2 level (in ppm)
1980	338.7	1992	356.4
1982	341.1	1994	358.9
1984	344.4	1996	362.6
1986	347.2	1998	366.6
1988	351.5	2000	369.4
1990	354.2	2002	372.9

Doğrusal Modeller

Notice that the data points appear to lie close to a straight line.

- So, in this case, it's natural to choose a linear model.

However, there are many possible lines that approximate these data points.

- So, which one should we use?

One possibility is the line that passes through the first and last data points.

The slope of this line is:

$$\frac{372.9-338.7}{2002-1980}=\frac{34.2}{22}\approx 1.5545$$

Doğrusal Modeller

- The equation of the line is: $C - 338.7 = 1.55(t - 1980)$ or $C = 1.55t - 2739$
- This equation gives one possible linear model for the CO_2 level. It is graphed in the figure.
- Although our model fits the data reasonably well, it gives values higher than most of the actual CO_2 levels.
- A better linear model is obtained by a procedure from statistics called linear regression.
- If we use a graphing calculator, we enter the data from the table into the data editor and choose the linear regression command.
- With Maple, we use the fit[leastsquare] command in the stats package.
- With Mathematica, we use the Fit command.

Doğrusal Modeller

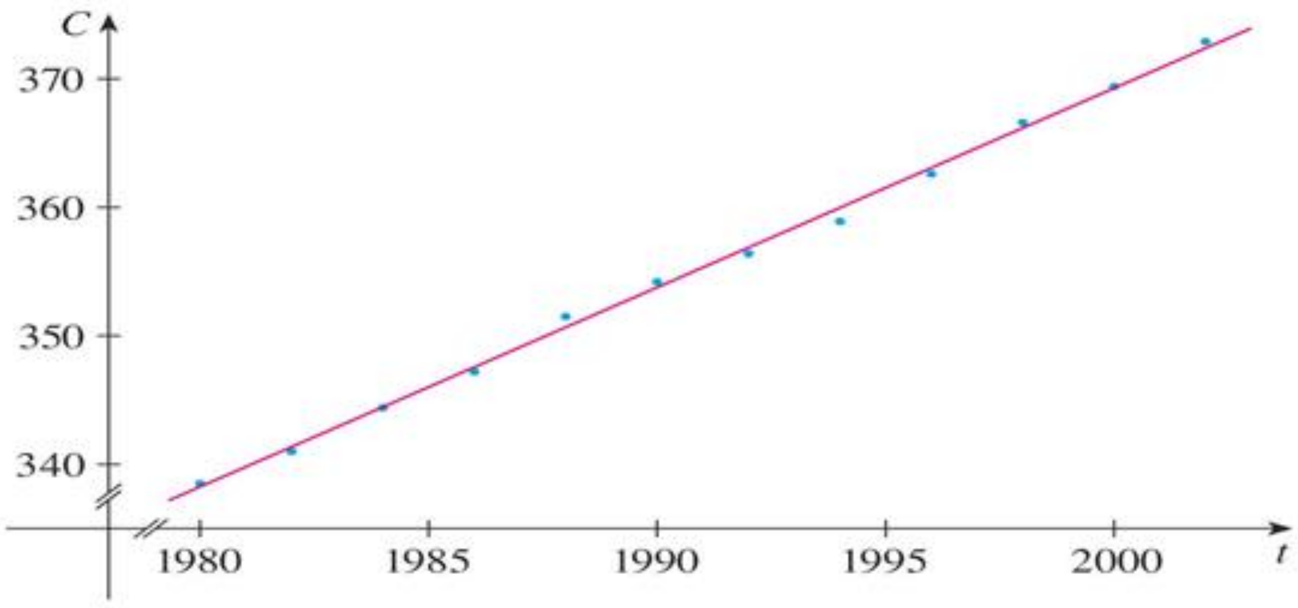
The machine gives the slope and *y*-intercept of the regression line as:

$m = 1.55$ $\qquad\qquad b = -2734$

So, our least squares model for the level CO_2 is: $C = 1.55t - 2734$

In the figure, we graph the regression line as well as the data points.

Comparing with the earlier figure, we see that it gives a better fit than our previous linear model.



Doğrusal Modeller

Use the linear model given by Equation 2 to estimate the average CO_2 level for 1987 and to predict the level for 2010.

- According to this model, when will the CO_2 level exceed 400 parts per million?
- Using Equation 2 with $t = 1987$, we estimate that the average CO_2 level in 1987 was: $C(1987) = (1.55)(1987) - 2734$
- This is an example of interpolation—as we have estimated a value between observed values.
- In fact, the Mauna Loa Observatory reported that the average CO_2 level in 1987 was 348.93 ppm.
- So, our estimate is quite accurate.

Doğrusal Modeller

With $t = 2010$, we get:

$C(2010) = (1.55)(2010) - 2734 \quad 384.81$

So, we predict that the average CO_2 level in 2010 will be 384.8 ppm.

- This is an example of extrapolation—as we have predicted a value outside the region of observations.
- Thus, we are far less certain about the accuracy of our prediction.

Using Equation 2, we see that the CO_2 level exceeds 400 ppm when: $1.55192t - 2734.55 > 400$

Solving this inequality, we get: $t > \dfrac{3134.55}{1.55192} \approx 2019.79$

- Thus, we predict that the CO_2 level will exceed 400 ppm by 2019.
- This prediction is somewhat risky—as it involves a time quite remote from our observations.

Polynomials

A function P is called a polynomial if P(x) = anxn + an-1xn-1 + … + a2x2 + a1x + a0

where n is a nonnegative integer and the numbers a0, a1, a2, …, an are constants called the coefficients of the polynomial.

The domain of any polynomial is ° $= (-\infty, \infty)$.

If the leading coefficient $a_n \neq 0$, then the degree of the polynomial is *n*.

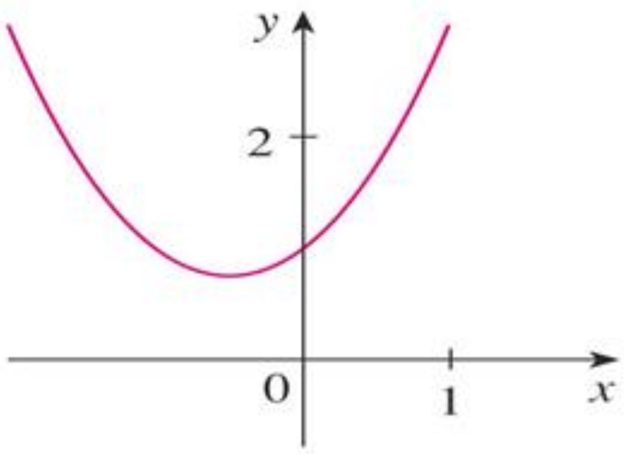
– For example, the function

$$P(x) = 2x^6 - x^4 + \frac{2}{5}x^3 + \sqrt{2}$$

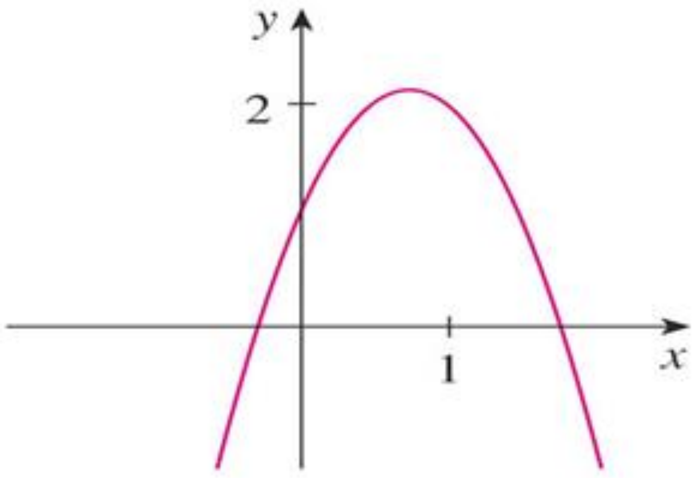
is a polynomial of degree 6.

Polynomials

- A polynomial of degree 1 is of the form $P(x) = mx + b$. So, it is a linear function.
- A polynomial of degree 2 is of the form P(x) = ax2 + bx + c. It is called a quadratic function.
- Its graph is always a parabola obtained by shifting the parabola y = x2. The parabola opens upward if a > 0 and downward



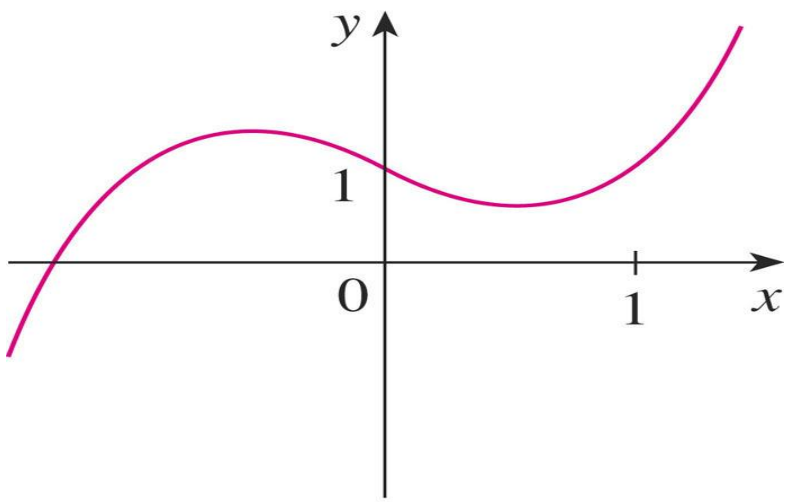
(a) $y = x^2 + x + 1$



(b) $y = -2x^2 + 3x + 1$

A polynomial of degree 3 is of the form

$$P(x) = ax^3 + bx^2 + cx + d \qquad (a \neq 0)$$



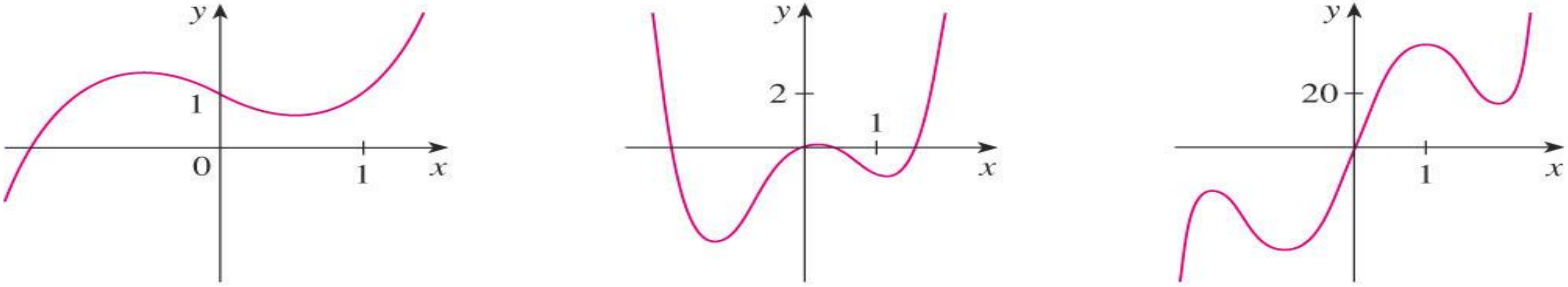
(a) $y = x^3 - x + 1$

It is called a cubic function.

POLYNOMIALS

We will see later why these three graphs have these shapes.

Polynomials are commonly used to model various quantities that occur in the natural and social sciences.

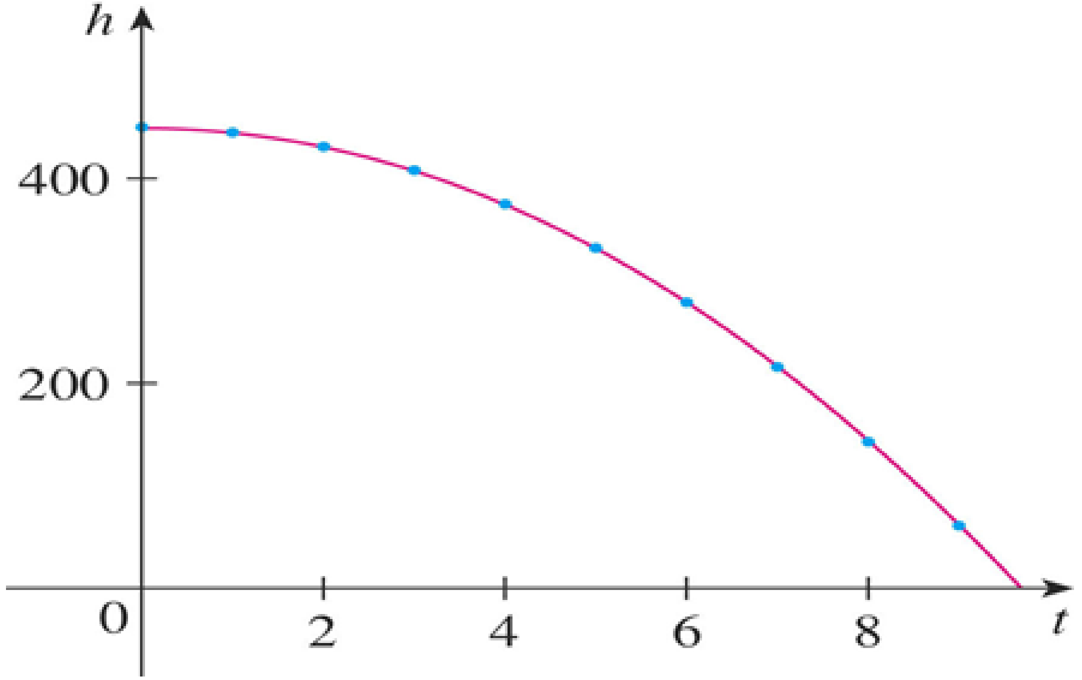


(a) $y = x^3 - x + 1$ (b) $y = x^4 - 3x^2 + x$ (c) $y = 3x^5 - 25x^3 + 60x$

POLYNOMIALS

A ball is dropped from the upper observation deck of the CN Tower—450 m above the ground—and its height *h* above the ground is recorded at 1-second intervals.

- Find a model to fit the data and use the model to predict the time at which the ball hits the ground.



Time (seconds)	Height (meters)
0	450
1	445
2	431
3	408
4	375
5	332
6	279
7	216
8	143
9	61

POLYNOMIALS

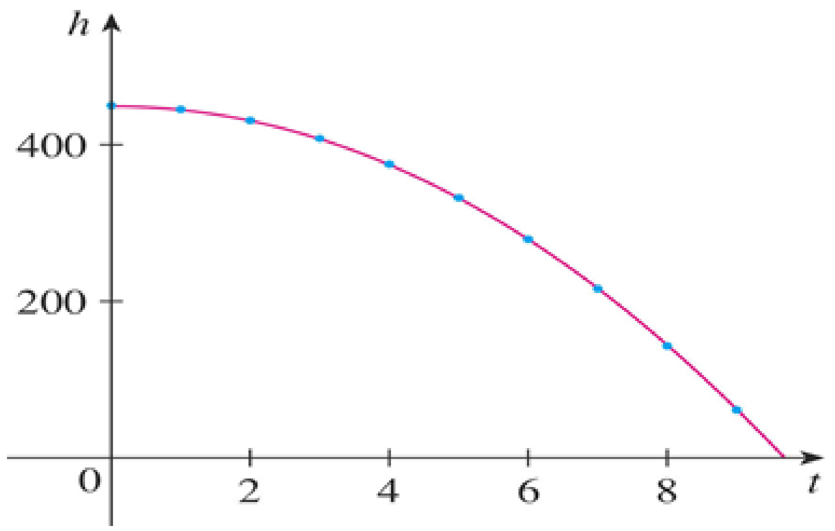
- We draw a scatter plot of the data. We observe that a linear model is inappropriate.
- However, it looks as if the data points might lie on a parabola. So, we try a quadratic model instead.
- Using a graphing calculator or computer algebra system (which uses the least squares method), we obtain the following quadratic model = $449.36 + 0.96t - 4.90t^2$
- We plot the graph of Equation 3 together with the data points. We see that the quadratic model gives a very good fit.

POLYNOMIALS

The ball hits the ground when h = 0. So, we solve the quadratic equation -4.90t2 + 0.96t + 449.36 = 0

The quadratic formula gives

$$t = \frac{-0.96 \pm \sqrt{(0.96)^2 - 4(-4.90)(449.36)}}{2(-4.90)}$$



– The positive root is $t \approx 9.67$

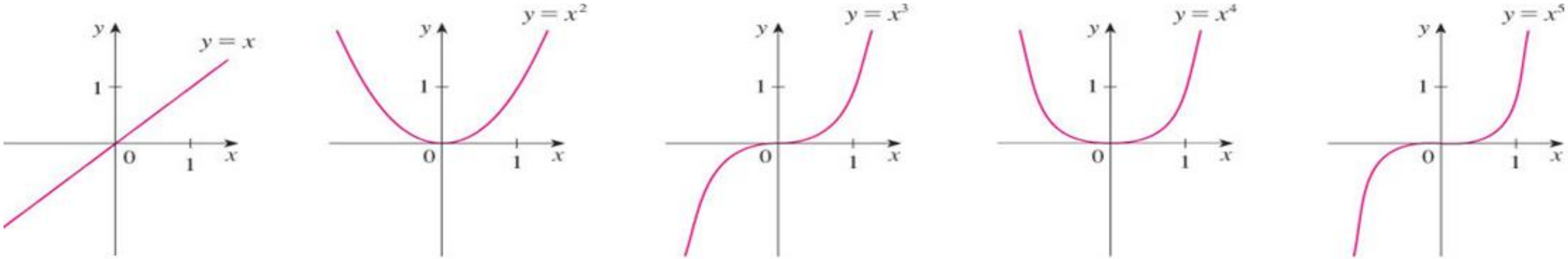
– So, we predict the ball will hit the ground after about 9.7 seconds.

POWER FUNCTIONS

A function of the form $f(x) = x^a$, where a is constant, is called a power function.

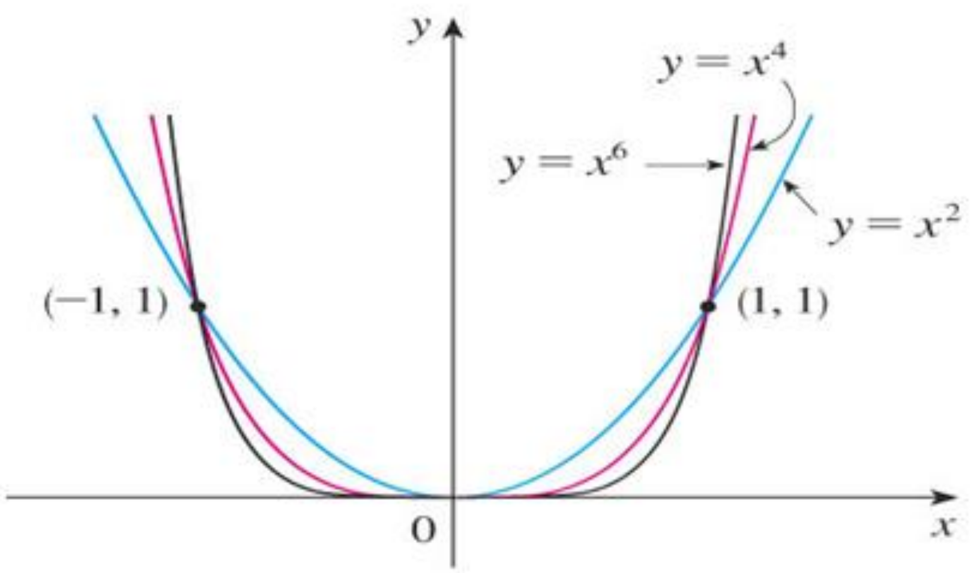
$a = n$, where n is a positive integer

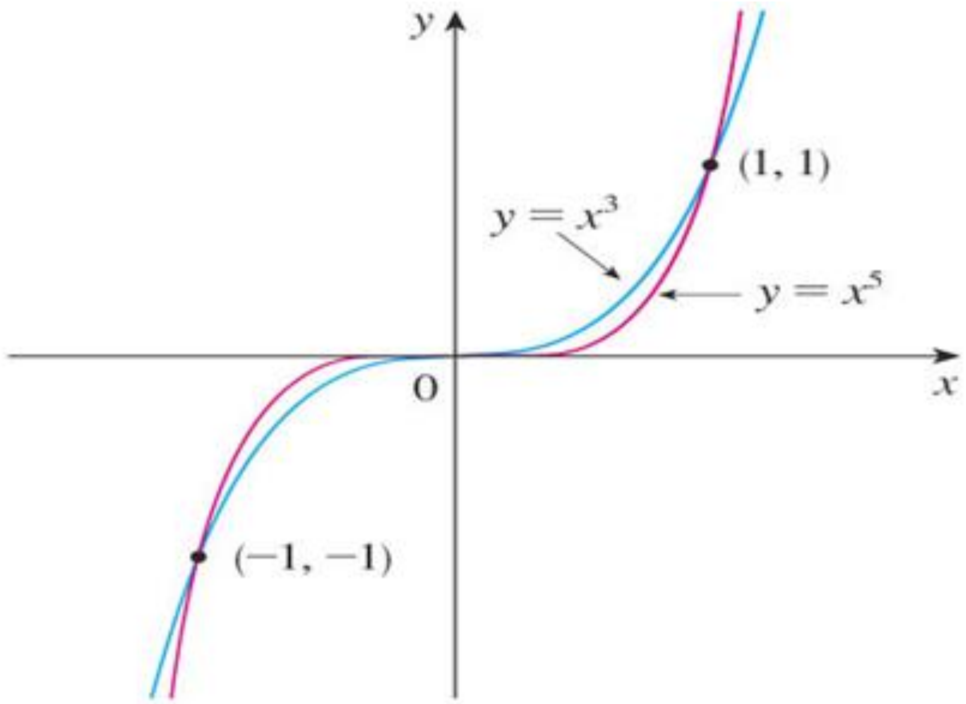
- The graphs of $f(x) = x^n$ for n = 1, 2, 3, 4, and 5 are shown.
- These are polynomials with only one term.
- We already know the shape of the graphs of $y = x$ (a line through the origin with slope 1) and $y = x^2$ (a parabola).



CASE

- The general shape of the graph of $f(x) = x^n$ depends on whether n is even or odd.
- If n is even, then $f(x) = x^n$ is an even function, and its graph is similar to the parabola $y = x^2$.
- If n is odd, then $f(x) = x^n$ is an odd function, and its graph is similar to that of $y = x^3$.
- However, notice from the figure that, as n increases, the graph of $y = x^n$ becomes flatter near 0 and steeper when $|x| \geq 1$. If x is small, then x^2 is smaller, x^3 is even smaller, x^4 is smaller still, and so on.

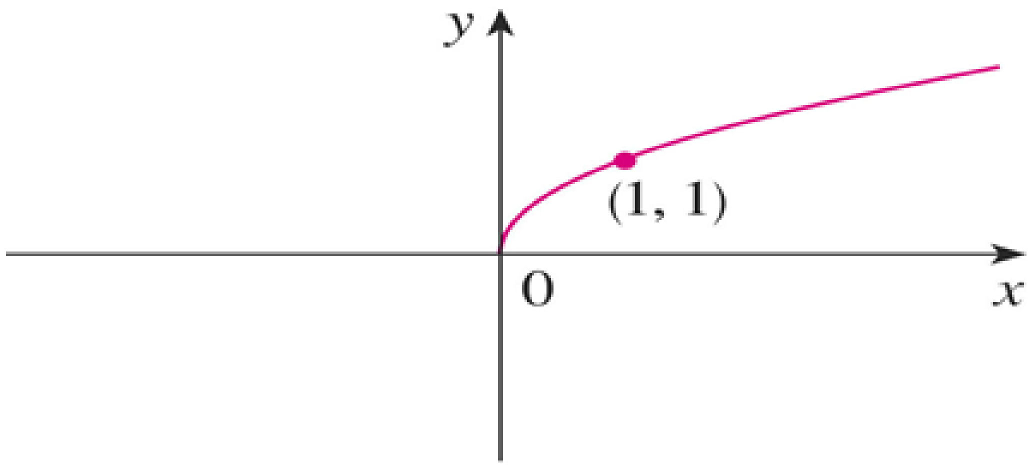




CASE

$a = 1/n$, where n is a positive integer

- The function $f(x) = x^{1/n} = \sqrt[n]{x}$ is a root function.
- For $n = 2$, it is the square root function $f(x) = \sqrt{x}$, whose domain is $[0, \infty)$ and whose graph is the upper half of the parabola $x = y^2$.
- For other even values of n, the graph of $y = \sqrt[n]{x}$ is similar to that of $y = \sqrt{x}$.

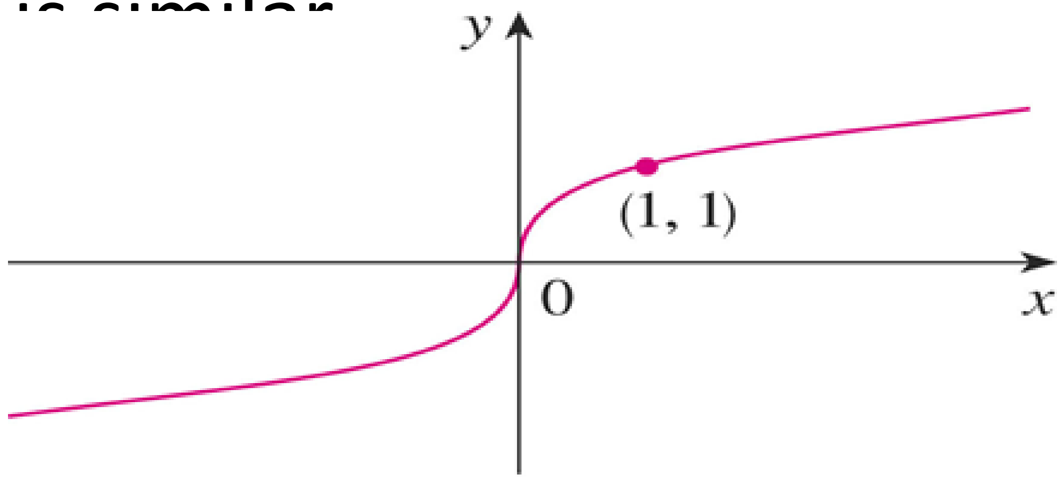


(a) $f(x) = \sqrt{x}$

CASE

For n = 3, we have the cube root function $f(x) = \sqrt[3]{x}$ whose domain is (recall that every real number has a cube root) and whose graph is shown. $y = \sqrt[n]{x}$

- The graph of $y = \sqrt[3]{x}$ for n odd (n > 3) is similar to that of .

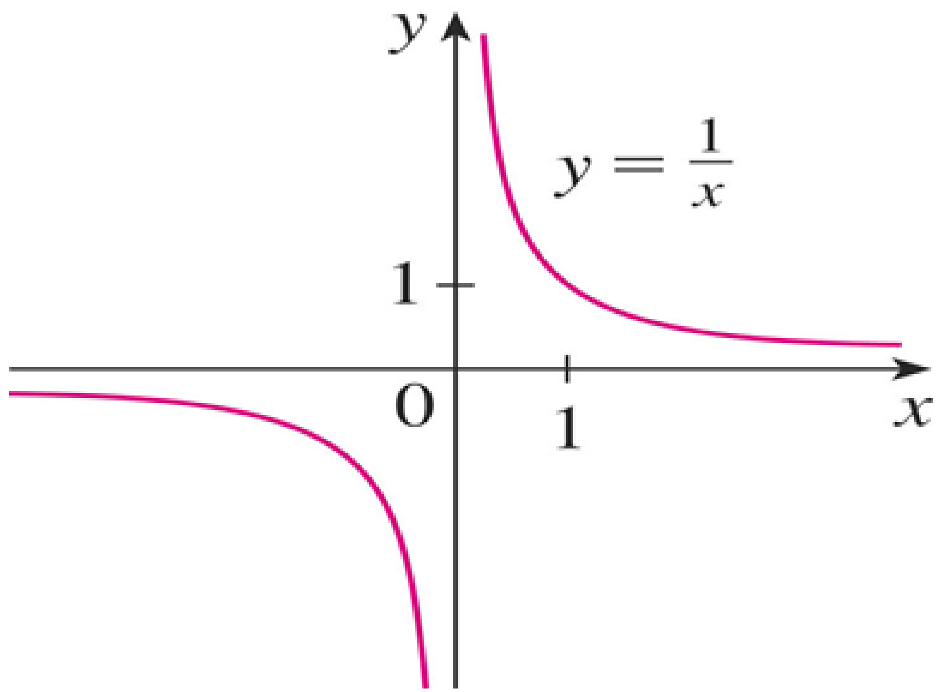


(b) $f(x) = \sqrt[3]{x}$

CASE

a = -1

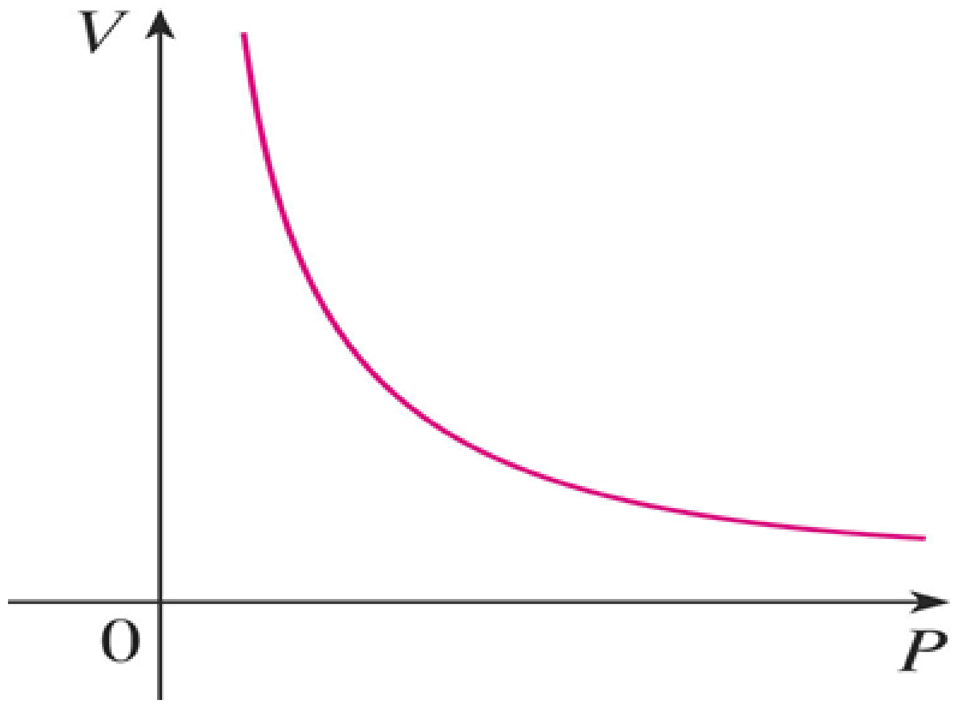
- The graph of the reciprocal function $f(x) = x^{-1} = 1/x$ is shown.
- Its graph has the equation $y = 1/x$, or $xy = 1$.
- It is a hyperbola with the coordinate axes as its asymptotes.

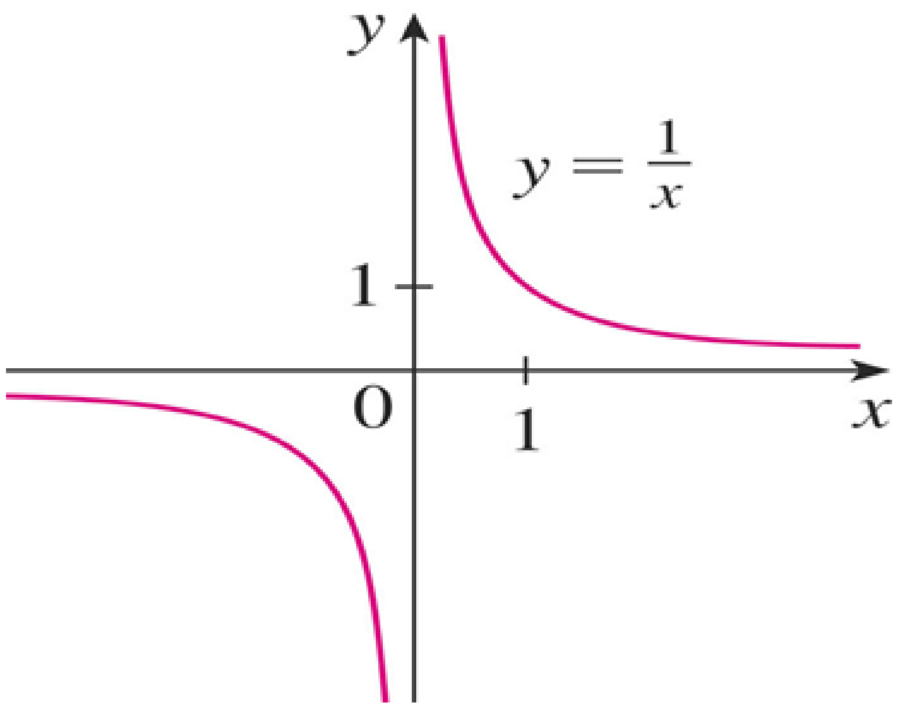


CASE

This function arises in physics and chemistry in connection with Boyle's Law, which states that, when the temperature is constant, the volume *V* of a gas is inversely proportional to the pressure *P*. $V=C/P$

where *C* is a constant. So, the graph of *V* as a function of *P* has the same general shape as the right half of the previous figure.





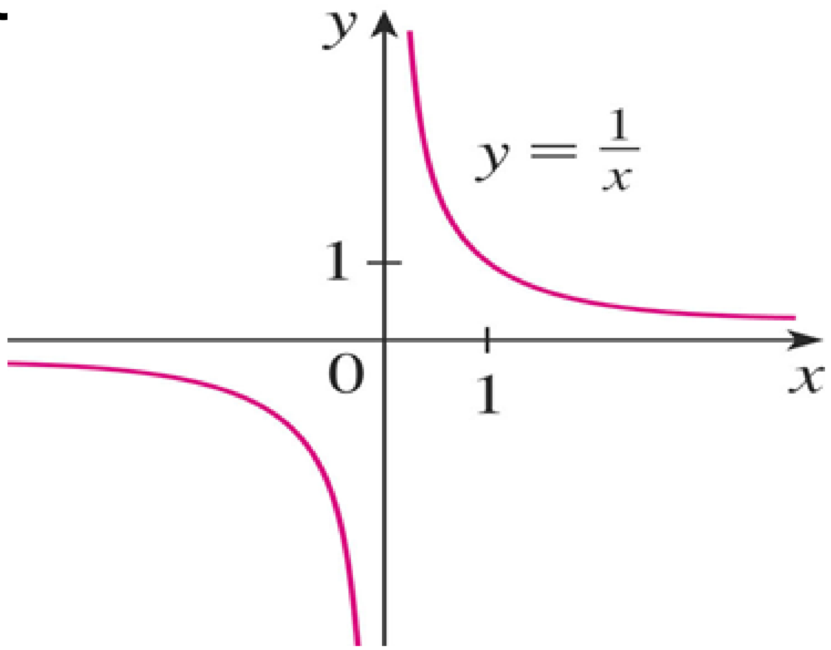
RATIONAL FUNCTIONS

A rational function *f* is a ratio of two polynomials $f\,(\mathrm{x}) = \dfrac{P(x)}{Q(x)}$

where *P* and *Q* are polynomials. The domain consists of all values of *x* such that $Q(x) \neq 0$.

A simple example of a rational function is the function $f(x) = 1/x$ whose domain is $\{\mathrm{x} | \mathrm{x} \neq 0\}$.

This is the reciprocal function graphed in the

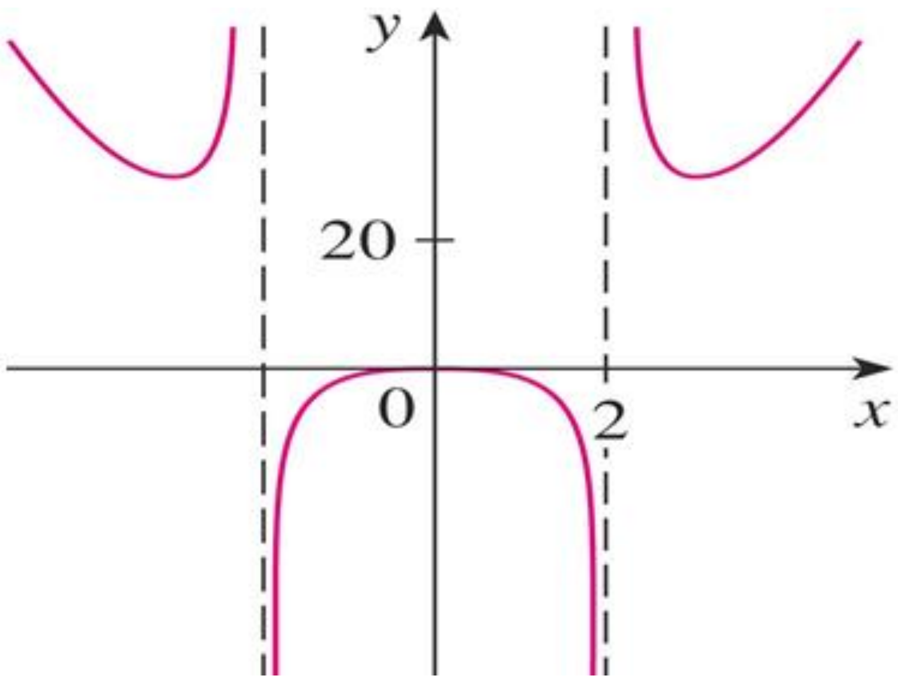


RATIONAL FUNCTIONS

The function

$$f(x) = \frac{2x^4 - x^2 + 1}{x^2 - 4}$$

is a rational function with domain $\{x \mid x \neq \pm 2\}$.



ALGEBRAIC FUNCTIONS

A function *f* is called an algebraic function if it can be constructed using algebraic operations—such as addition, subtraction, multiplication, division, and taking roots—starting with polynomials.

Any rational function is automatically an algebraic function.

Here are two more examples:

$$f(x) = \sqrt{x^2 + 1}$$

$$g(x) \frac{x^4 - 16x^2}{x + \sqrt{x}} + (x - 2)\sqrt[3]{x + 1}$$

ALGEBRAIC FUNCTIONS

An example of an algebraic function occurs in the theory of relativity.

– The mass of a particle with velocity v is

$$m = f(v) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

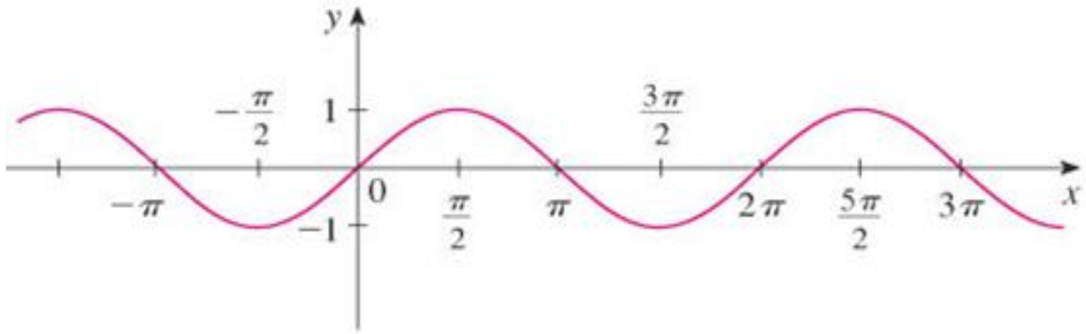
where m_0 is the rest mass of the particle and $c = 3.0 \times 10^5$ km/s is the speed of light in a vacuum.

TRIGONOMETRIC FUNCTIONS

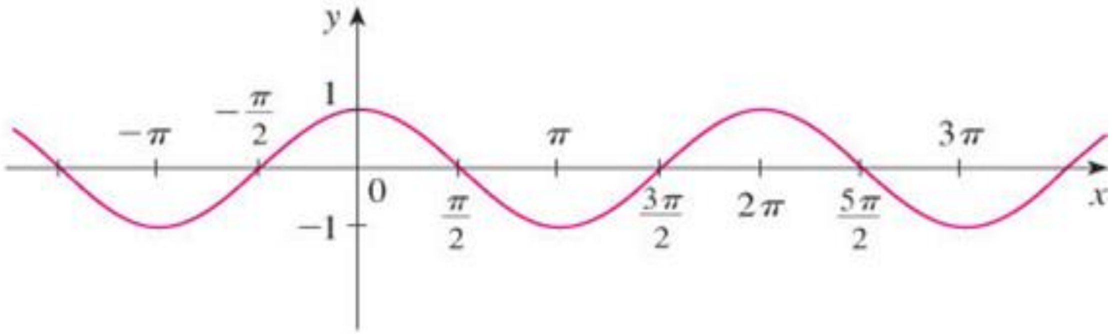
In calculus, the convention is that radian measure is always used (except when otherwise indicated).

– For example, when we use the function $f(x) = \sin x$, it is understood that $\sin x$ means the sine of the angle whose radian measure is x.

– Thus, the graphs of the sine and cosine functions are as shown in the figure.



(a) $f(x) = \sin x$



(b) $g(x) = \cos x$

TRIGONOMETRIC FUNCTIONS

- Also, the zeros of the sine function occur at the integer multiples of π . That is, $\sin x = 0$ when $x = n\pi$, n an integer.
- An important property of the sine and cosine functions is that they are periodic functions and have a period 2π. This means that, for all values of x, $\sin(x + 2\pi) = \sin(x)$, $\cos(x + 2\pi) = \cos(x)$.
- Notice that, for both the sine and cosine functions, the domain is $(-\infty, \infty)$ and the range is the closed interval [-1, 1]. Thus, for all values of x, we have: $-1 \leq \sin(x) \leq 1$, $-1 \leq \cos(x) \leq 1$. In terms of absolute values, it is: $|\sin(x) \leq 1|$, $|\cos(x) \leq 1|$.

TRIGONOMETRIC FUNCTIONS

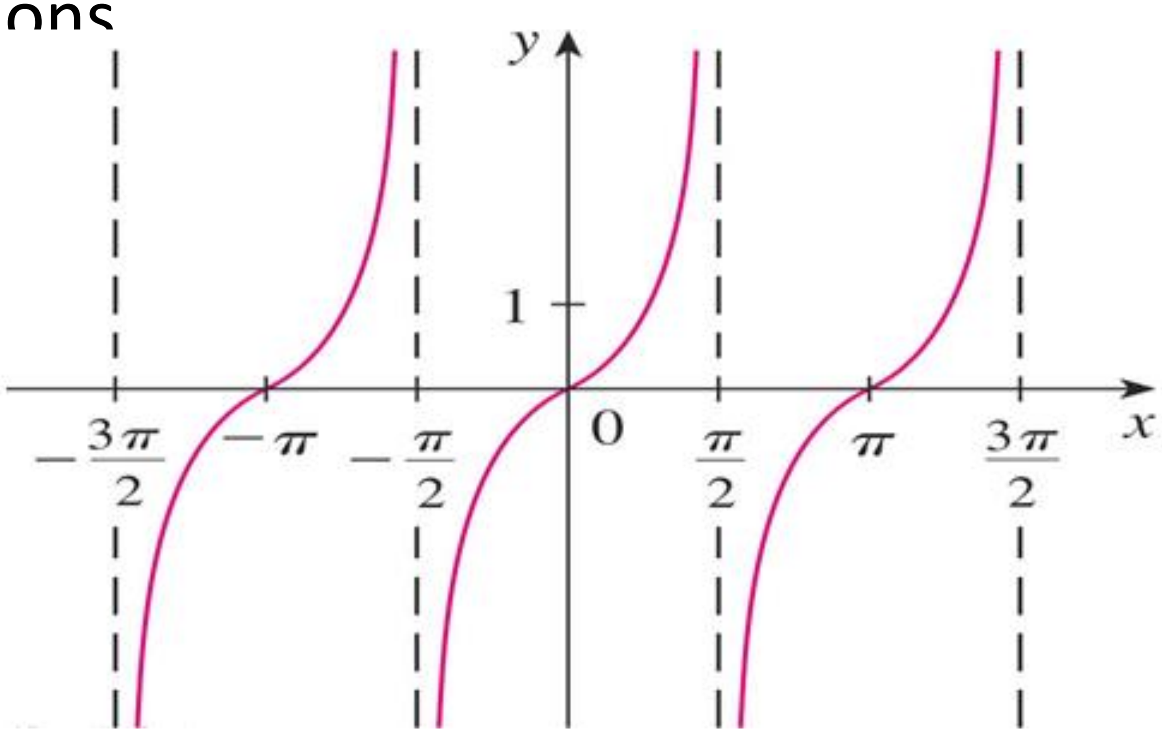
The periodic nature of these functions makes them suitable for modeling repetitive phenomena such as tides, vibrating springs, and sound waves.

For instance, in Example 4 in Section 1.3, we will see that a reasonable model for the number of hours of daylight in Philadelphia *t* days after January 1 is given by the function:

$$L(t) = 12 + 2.8 \sin\left[\frac{2\pi}{365}(t - 80)\right]$$

TRIGONOMETRIC FUNCTIONS

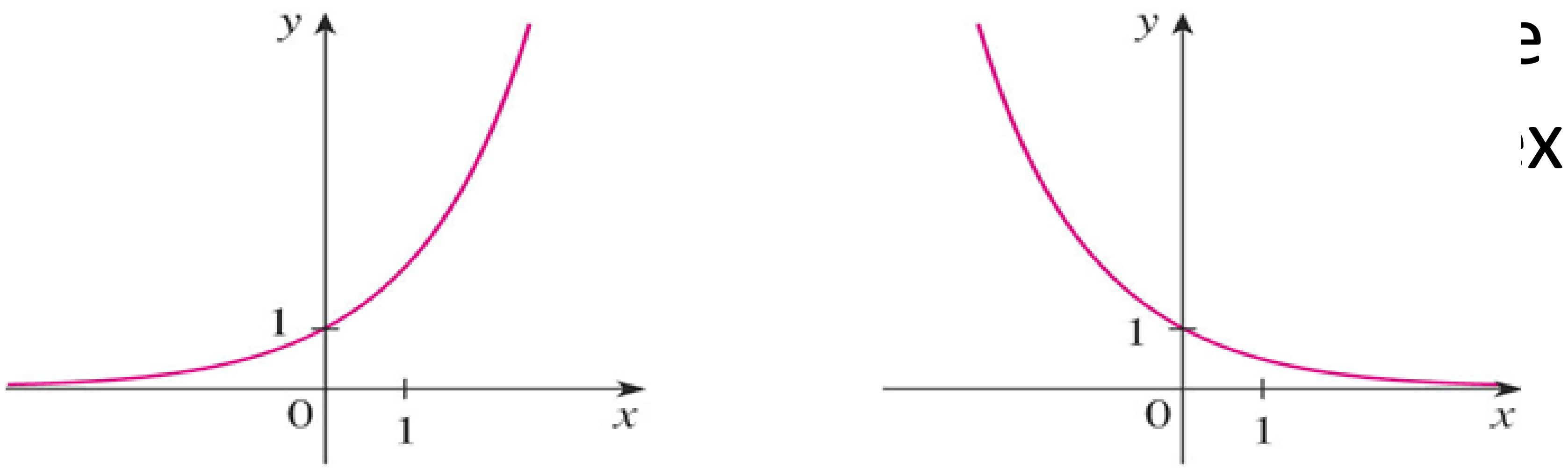
- The tangent function is related to the sine and cosine functions by the equation $\tan(\mathrm{x}) = \frac{sin(\mathrm{x})}{\cos(\mathrm{x})}$
- The tangent function is undefined whenever cos *x* = 0, that is, when $\mathrm{x} = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
- Its range is $(-\infty, \infty)$. Notice that the tangent π function has period: tan(x+π) = tan(x) for all *x.*
- The remaining three trigonometric functions—cosecant, secant, and cotangent—are the reciprocals of the sine, cosine, and tangent functions.



EXPONENTIAL FUNCTIONS

The exponential functions are the functions of the form $f(x)=a^x$, where the base a is a positive constant.

- The graphs of $y = 2^x$ and $y = (0.5)^x$ are shown.
- In both cases, the domain is $(-\infty, \infty)$ and the range is $(0, \infty)$.
- We will see that they are useful for modeling many natural phenomena—such as population growth (if $a > 1$) and radioactive decay (if $a < 1$).
- The log … e base a is a positive … xponential functio



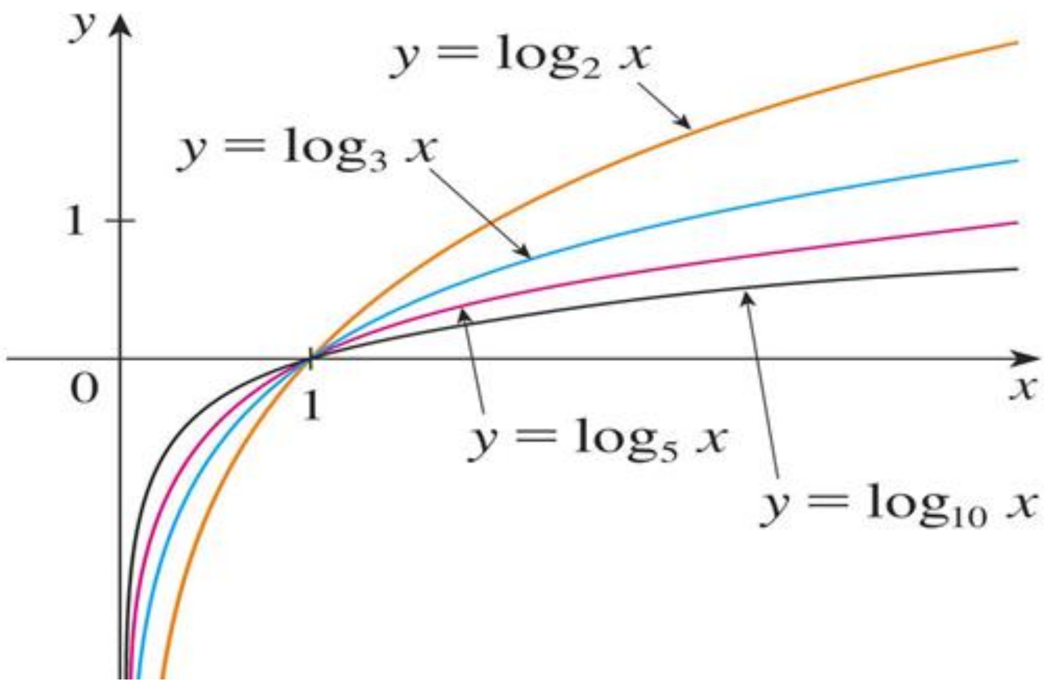
(a) $y = 2^x$

(b) $y = (0.5)^x$

LOGARITHMIC FUNCTIONS

The figure shows the graphs of four logarithmic functions with various bases.

- In each case, the domain is $(0, \infty)$, the range i $(-\infty, \infty)$, and the function increases slowly when x > 1.



TRANSCENDENTAL FUNCTIONS

Classify the following functions as one of the types of functions that we have discussed.

- $f(x) = 5^x$ is an exponential function. The x is the exponent
- $g(x) = x^5$ is a power function. The x is the base. We could also consider it to be a polynomial of degree 5.
- $u(t) = 1 - t + 5t^4$ is a polynomial of degree 4.

This is an algebraic function.

$$h(x) = \frac{1+x}{1-\sqrt{x}}$$

Transcendental functions are those that are not algebraic.

- The set of transcendental functions includes the trigonometric, inverse trigonometric, exponential, and logarithmic functions.
- However, it also includes a vast number of other functions that have never been named.

Kaynaklar

- Analog Electronics, Bilkent Unıversity
- Electric Circuits Ninth Edition, James W. Nilsson Professor Emeritus Iowa State University, Susan A. Riedel Marquette University, Prentice Hall, 2008.
- Lessons in Electric Circuits, By Tony R. Kuphaldt Fifth Edition, last update January 10, 2004.
- Fundamentals of Electrical Engineering, Don H. Johnson, Connexions, Rice University, Houston, Texas, 2016.
- Introduction to Electrical and Computer Engineering, Christopher Batten - Computer Systems Laboratory School of Electrical and Computer Engineering, Cornell University, ENGRG 1060 Explorations in Engineering Seminar, Summer 2012.
- Introduction to Electrical Engineering, Mulukutla S. Sarma, Oxford University Press, 2001.
- Basics of Electrical Electronics and Communication Engineering, K. A. NAVAS Asst.Professor in ECE, T. A. Suhail Lecturer in ECE, Rajath Publishers, 2010.
- İnternet ortamından sunum ve ders notları

Usage Notes

- These slides were gathered from the presentations published on the internet. I would like to thank who prepared slides.
- Also, these slides are made publicly available on the web for anyone to use
- If you choose to use them, I ask that you alert me of any mistakes which were made and allow me the option of incorporating such changes (with an acknowledgment) in my set of slides.

Sincerely,

Dr. Cahit Karakuş

cahitkarakus@gmail.com

thank you